# THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 2 March 1, 1932 No. 242

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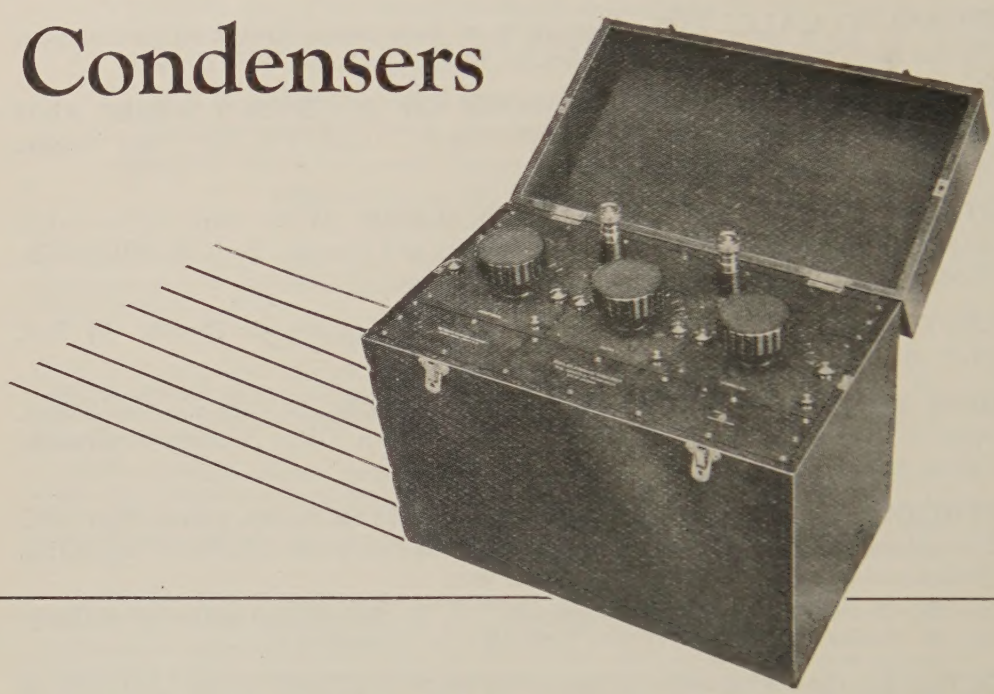

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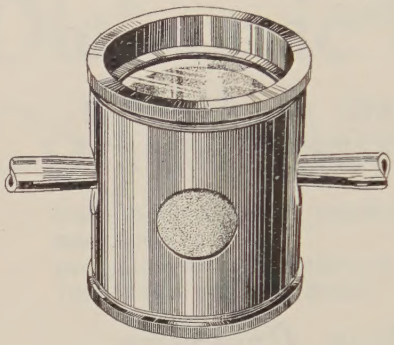

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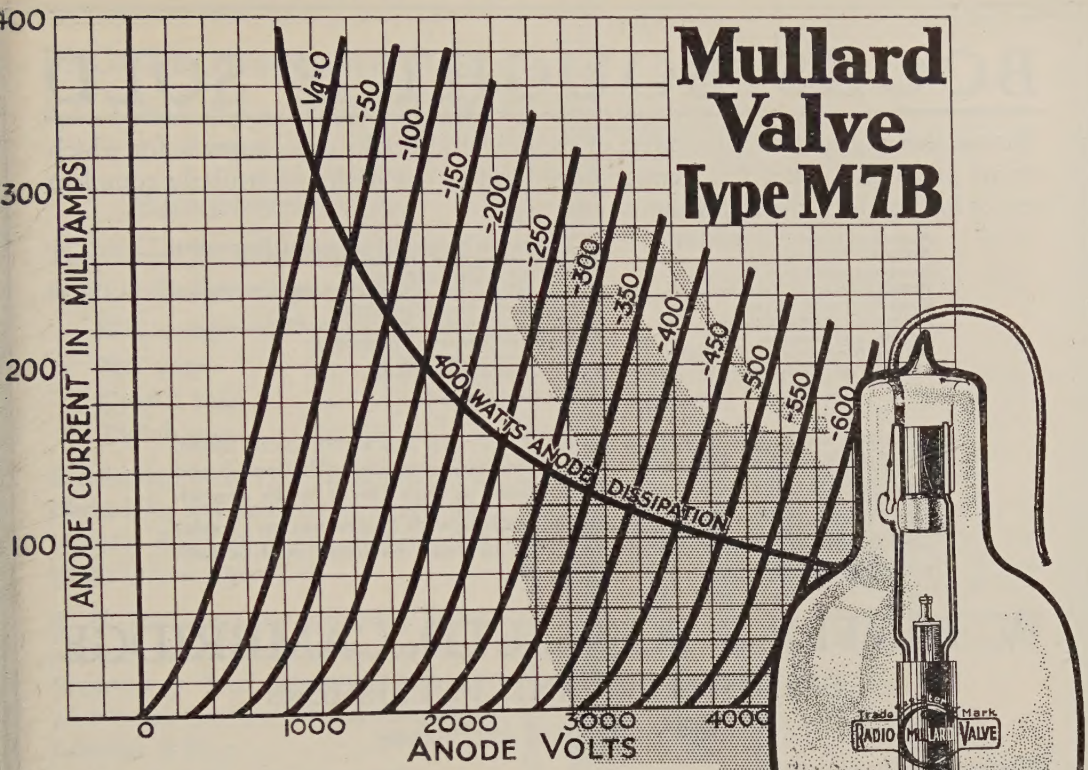


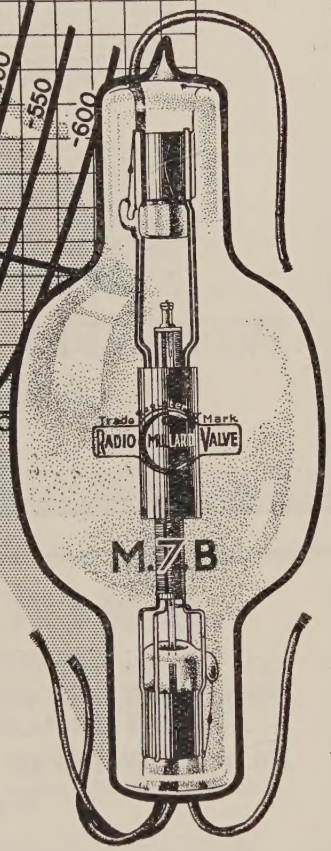


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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 2 *March* 1, 1932 NO. 242

# THE SPECIFIC VOLUMES OF SOME GASEOUS REFRIGERANTS

BY J. H. AWBERY, B.A., B.Sc., F.INST.P., AND EZER GRIFFITHS, D.Sc., F.R.S.

Physics Department, National Physical Laboratory

ERRATUM

*Proc. Phys. Soc.* **44**, 106 (1932). Review of *The Constitution of Atomic Nuclei and Radio-activity*. The author is G. Gamow and the publisher Humphrey Milford.

## § 1. INTRODUCTION

THE specific volume, i.e. the volume of 1 gm. of vapour at a given temperature and the corresponding vapour pressure, is a quantity which assumes importance in considering the utility of a substance as a refrigerant, since it determines the size of compressor necessary for a given duty of the refrigerating plant.

## § 2. EXPERIMENTAL METHOD

The method we have adopted for this determination is in principle a very simple one. The liquid is contained in a vessel which communicates through a valve with a second vessel which can be exhausted, and the whole apparatus is immersed in a bath controlled thermostatically. Thus, if the valve is opened when temperature equilibrium has been reached, vapour from the liquid expands and fills the second vessel at the appropriate vapour pressure. After closing the valve the second vessel may be exhausted again and the process commenced anew. Thus the measurement consists essentially in filling this vessel with vapour a counted number of times, and determining the amount of liquid used up in the process; this latter quantity is found by weighing the first vessel (the liquid container) before and after the experiments.

The actual apparatus is shown in figure 1, where the vessel containing the

liquid is seen on the left. It is made of brass and entirely sealed up, and contains two re-entrant tubes, one for a thermometer and one for a heater. The heater is provided for accelerating the process of re-establishing temperature equilibrium with the bath, after the cooling which accompanies the evaporation when the valve is opened. It is essential to stir the liquid, and the problem of doing so without the possibility of loss or leakage was overcome by operating the stirrer electromagnetically. A coil was slipped over the neck of the vessel, and current could be sent through it by closing a circuit with a tapping key. The stirrer consisted of a brass plate with a number of holes in it, attached to a brass rod carrying a piece of soft iron at the top. Thus when the circuit was completed the stirrer was lifted, and by repeated tappings the stirrer could be oscillated up and down.

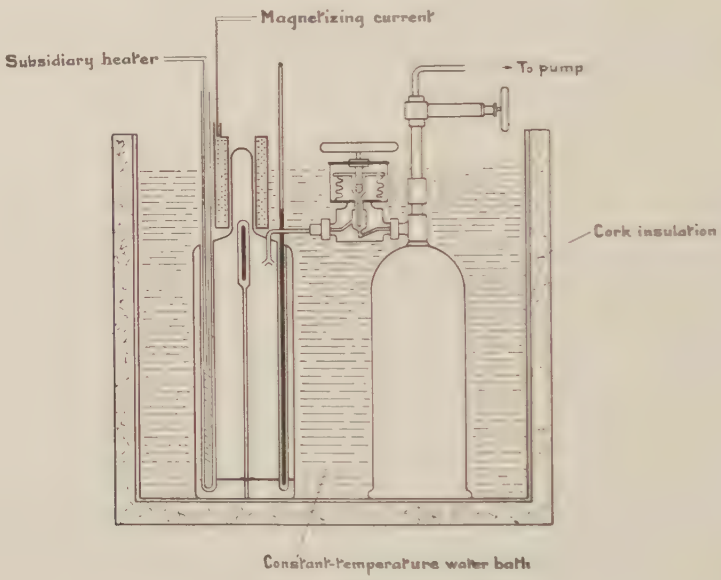


Fig. 1. Section through apparatus.

The construction of the valve between the two vessels may be seen in the diagram, in which its size is somewhat exaggerated in order to show the details. Since sulphur dioxide was one of the substances to be studied, it was necessary to avoid all the usual packing materials; this was carried out by surrounding the upper part of the valve with flexible metal tubing having corrugated walls, soldered to the base and to the upper plate seen in the figure. By this construction it will be seen that no gas can escape through the valve.

The vessel for receiving the vapour was of glass, so that the presence of condensed liquid could be detected if the temperature changed at any time. It was provided with a valve through which connexion could be made to the pump used in exhausting the vessel. This valve was a needle valve with an exceptionally long and fine screw thread on the stem. The volume of this vessel was 1270 cm.$^3$, determined by weighing it full of water.

## § 3. RESULTS

Two corrections are required for each observation. After being exhausted the second vessel still contains a small quantity of vapour, corresponding to the pressure down to which the pump exhausts, so that the effective volume is reduced for the next filling. The details of a typical experiment will show how this correction is made, and will illustrate the order of magnitude of the quantities involved. The second correction is for the weight of vapour which remains in the first vessel, to fill the space originally occupied by the liquid which has been evaporated.

*A typical experiment: temperature* 0° *C., sulphur dioxide.* Twelve expansions were carried out and the decrease in weight of the first vessel was 73·0 gm. The volume of the expansion vessel was 1270 cm.$^3$ When the latter was first exhausted, the residual gas was air, which by Dalton's law does not affect the quantity of $SO_2$ required to fill the vessel at any given pressure. The mean of the residual pressures in the remaining expansions was 8 mm. of mercury, the barometer being about 760 mm. Thus the effective volume was 752/760 of the true volume, or the weight may be increased in the ratio 760/752. Thus the weight corrected for this effect is 73·8 gm. For the second correction, we know that the density of the liquid is of the order 1·4, so that 73·8/1·4 or 53 cm.$^3$ have been removed. Since the density of the vapour is approximately 73·8/12 × 1270 gm. per cm.$^3$ (from the experiment itself), the mass of vapour left in the vessel and included in the weighing is about 53 × 73·8/12 × 1270 or 0·3 gm., and this should be added to the observed loss in weight. Thus the weight finally corrected is 74·1 gm. and the specific volume is 12 × 1270/74·1, or 206 cm.$^3$/gm. at 0° C.

The remaining results for $SO_2$ are set out in table 1, and shown in figure 2.

Table 1. Results for $SO_2$.

| Temperature (° C.) | Specific volume (cm.$^3$/gm.) | Density of saturated vapour (gm./cm.$^3$) |
|---|---|---|
| 0 | 206 | 0·00485 |
| 0 | 218 | 0·00459 |
| 13·4 | 134 | 0·00749 |
| 23·6 | 88 | 0·01133 |

The $SO_2$ used had a moisture content of 0·3 per cent. in the liquid phase and 0·1 per cent. in the gaseous.

*Dichloroethylene.* This material, which corresponds chemically to three different substances, was purchased from a firm of manufacturing chemists and appeared to consist of a mixture of organic compounds. It had no definite boiling-point, but after two previous fractionations it was found that:

6 per cent. distilled below 49° C.
22 ,, ,, between 49° and 54° C.
8 ,, ,, between 54° and 55° C.
13 ,, ,, between 55° and 56° C.

16 per cent. distilled between 56° and 57° C.
22 „ „ between 57° and 58° C.
11 „ „ between 58° and 58·6° C.
2 „ „ above 58·6° C.

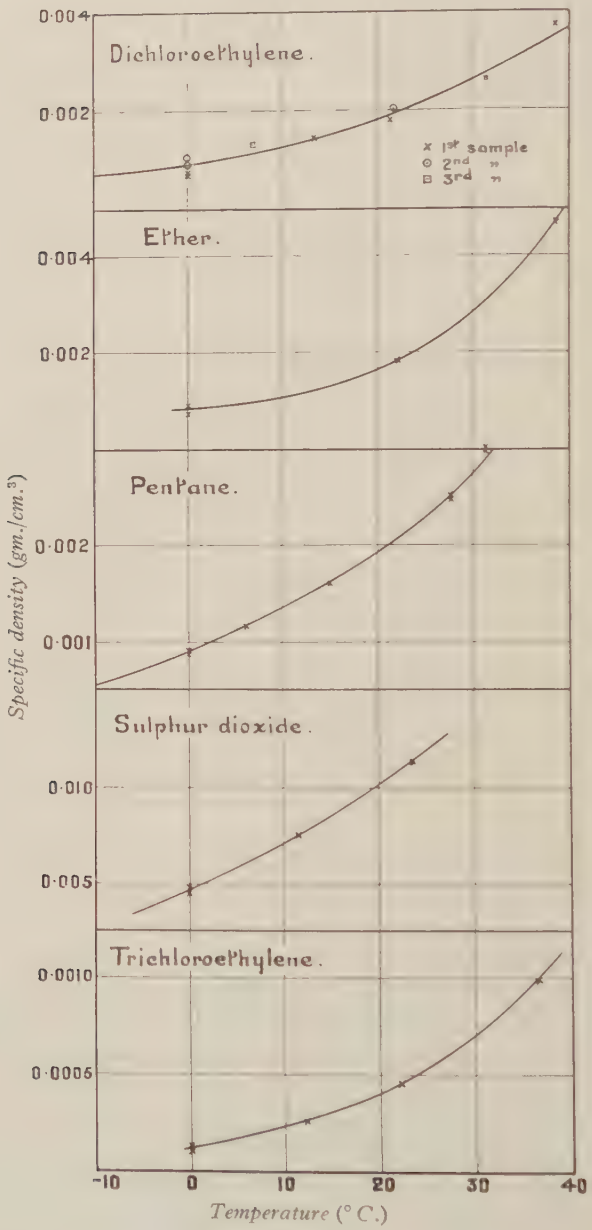


Fig. 2. Results.

Samples taken from different bottles of similar material did not give identical results. They are given in figure 2 and table 2.

Table 2. Results for dichloroethylene.

| Temperature (° C.) | Bottle | Specific volume (cm.³/gm.) | Density of saturated vapour (gm./cm.³) |
|---|---|---|---|
| 0 | A | 1350 | 0·00074 |
| 0 | A | 1390 | 0·00072 |
| 0 | B | 1110 | 0·00090 |
| 0 | B | 960 | 0·00104 |
| 7·0 | C | 758 | 0·00132 |
| 13·4 | A | 694 | 0·00144 |
| 21·4 | A | 552 | 0·00181 |
| 21·8 | B | 490 | 0·00204 |
| 31·5 | C | 379 | 0·00264 |
| 38·8 | A | 266 | 0·00376 |

*Ether.* It was thought to be of interest to determine the specific volume of commercial "sulphuric ether." The results are given in figure 2 and table 3.

Table 3. Results for commercial "sulphuric ether."

| Temperature (° C.) | Specific volume (cm.³/gm.) | Density of saturated vapour (gm./cm.³) |
|---|---|---|
| 0 | 1350 | 0·00074 |
| 0 | 1010 | 0·00099 |
| 22·1 | 540 | 0·00185 |
| 38·7 | 211 | 0·00473 |

*Pentane.* Pentane from a commercial source of supply was used, and appeared to be a mixture of hydrocarbons containing both iso-pentane and normal pentane. The results are given in figure 2 and table 4.

Table 4. Results for pentane

| Temperature (° C.) | Specific volume (cm.³/gm.) | Density of saturated vapour (gm./cm.³) |
|---|---|---|
| 0 | 1140 | 0·00088 |
| 0 | 1110 | 0·00090 |
| 6·1 | 870 | 0·00115 |
| 15·0 | 630 | 0·00159 |
| 25·7 | 403 | 0·00248 |
| 25·8 | 397 | 0·00252 |
| 31·4 | 334 | 0·00299 |
| 31·4 | 333 | 0·00300 |
| 31·4 | 332 | 0·00301 |

*Trichloroethylene*. The results with this material are shown in figure 2 and table 5.

Table 5. Results for trichloroethylene.

| Temperature (° C.) | Specific volume ($cm.^3/gm.$) | Density of saturated vapour ($gm./cm.^3$) |
|---|---|---|
| 0 | 8850 | 0·000113 |
| 0 | 7630 | 0·000131 |
| 12·1 | 3940 | 0·000254 |
| 22·1 | 2220 | 0·000450 |
| 36·7 | 1010 | 0·000990 |

## § 4. ACKNOWLEDGMENTS

This work was carried out for the Food Investigation Board, to whom we are indebted for permission to publish these results. Our thanks are due to Mr T. I. Jones, B.Sc., A.R.C.S., D.I.C., for bringing to our notice the principle employed, and to the Superintendent of the Physics Department, for facilities for the work. Mr A. Snow, Observer, assisted in the design of the apparatus, whilst we have to thank Mr A. R. Challoner, Senior Observer, for his patient assistance in taking readings.

## DISCUSSION

For discussion see page 126.

# THE LATENT HEAT OF SOME REFRIGERANTS

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*ABSTRACT.* The latent heats of evaporation of ethyl chloride, methyl chloride, sulphur dioxide, pentane and dichloroethylene have been measured over temperature ranges lying between − 18° C. and + 25° C. The method used is an electrical one in which a known amount of energy is supplied at constant temperature, and the mass evaporated is obtained by weighing.

## § 1. INTRODUCTION

In refrigeration certain fluids are commonly used for the cycle, and suitable types of machine have been evolved for the compression of the vapours of these fluids. In large-scale work, ammonia and carbon dioxide are the two refrigerants in general use. For land work ammonia is largely favoured because the pressures involved in the refrigerating cycle never greatly exceed 12 atmospheres. In the event of leaks, the gas not only has the disadvantage of being noxious to human beings, but also is said to reduce the commercial value of any foodstuffs with which it comes into contact. The first disadvantage is so great in the confined space of a ship that carbon dioxide, despite the higher pressures necessary (up to 70 atmospheres), is very generally used, and particularly so when the refrigerating plant is located in the main engine-room. On account of their wide use, the properties of these two fluids have been extensively studied* and their physical constants may now be considered as well determined.

There have, however, in recent years, been suggestions from various quarters that other organic and inorganic fluids might prove to be more suitable refrigerants for special requirements. The data available on the physical properties of these suggested fluids are very meagre, and some years ago we searched the available literature and compiled tables based on the best estimates we could then make for a number of properties, including the latent heat†. In the absence of direct determinations this had frequently to be calculated thermodynamically from the vapour pressure and the volume of the vapour, the latter having sometimes to be estimated by Boyle's law from the standard density, or from the assumption that a gram-molecule occupies 22·4 litres at normal temperature and pressure. It was from the

* *Proc. I. Mech. E.* 639 (1914) ($CO_2$); *Phil. Trans.* A, **215**, 215 (1915) ($CO_2$); *Bureau of Stds.* Bulletin No. 14 (1918) ($NH_3$).

† *Proc. British Cold Storage and Ice Association*, **21**, 63 (1925).

first clear that these tentative values would require revision and we have now measured the latent heat and the specific volume of the saturated vapours of some of the substances in question. The present paper is devoted to the latent-heat determinations.

## § 2. DESCRIPTION OF APPARATUS

The apparatus is shown in figure 1. In principle, the method is to evaporate the liquid in a calorimeter enclosed in a constant-temperature air bath, by supplying heat to it electrically. By this device heat-loss corrections are eliminated, since if the rate of electrical supply is suitably adjusted the liquid boils at the constant temperature, and all the energy dissipated is used in supplying the latent heat necessary for the evaporation. To look at the matter in a slightly different way,

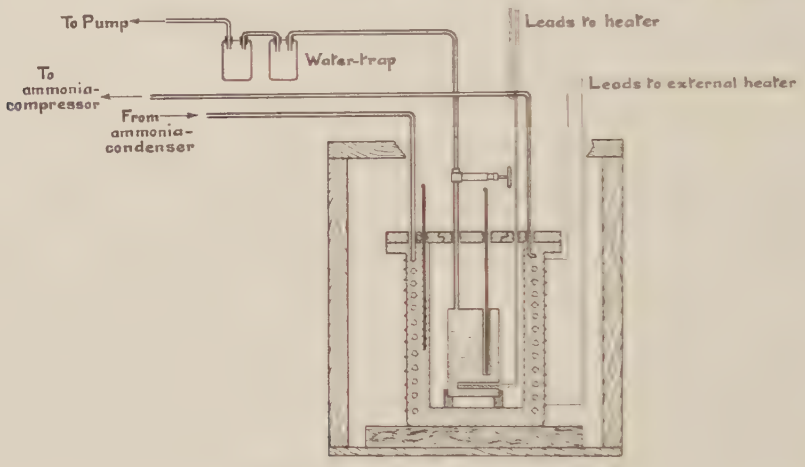



Fig. 1. Section of Apparatus.

it is clear that if the valve were opened, with no electric current flowing, the escaping vapour would abstract heat from the liquid and so lower the temperature of the calorimeter. The electrical supply is adjusted so that this fall of temperature is exactly compensated and there is a balance between the (electrical) supply and the energy carried off as latent heat.

The calorimeter proper consists of a steel vessel about 10 cm. high and 9 cm. in diameter, so that it holds nearly a litre of liquid. Two closed-end steel tubes project into the calorimeter (see figure 1). Of these, the vertical one was used for a thermometer and the horizontal one for the heater, which consisted of platinum wire wound on mica. The resistance of the heater was about 20 ohms. A narrow-bore steel tube projecting from the top of the calorimeter leads to a valve which is situated above the lid of the constant-temperature enclosure. All the joints in the calorimeter are welded.

The calorimeter was placed within the constant-temperature enclosure, and insulated from it by means of three boxwood pegs. The enclosure itself was made by casting aluminium around a spiral of iron piping, so as to make an aluminium

vessel about 25 cm. in diameter and about 30 cm. high, with the piping embedded in its walls. Thus by the use of the spiral as the evaporator of a refrigerating plant, the whole of the metal was kept at a uniform low temperature which was measured by means of a thermometer in a thermometer well cast in the walls of enclosure. The lid of the enclosure was also of aluminium, and was made in two halves with three holes to take the thermometer, current and potential leads, and the tube carrying the valve. A heating coil was also wound around the outside of the aluminium enclosure, to give a finer control than would be obtained with the refrigerating plant, and also for maintaining the enclosure at temperatures above atmospheric when required. The whole enclosure was further surrounded by cork in a box, deep enough to enclose also the valve from the calorimeter.

For liquids of high boiling-point, whose rate of evaporation at atmospheric pressure was very slow, a filter pump was provided so that the pressure in the apparatus could be reduced to that corresponding to the vapour pressure of the liquid at the temperature of experiment.

## § 3. METHOD OF CONDUCTING AN EXPERIMENT

The enclosure and all the apparatus having been allowed to reach a steady temperature, the valve was opened and the suction pump was started if necessary. In the earlier experiments, the current also was switched on at the same instant, but later it was found more convenient to defer this for one minute. Readings of the current in and the e.m.f. across the heater were taken with an ammeter and voltmeter. The temperature in the calorimeter was carefully watched, and the current was adjusted, when necessary, at times which were noted accurately, so as to keep the mean calorimeter temperature equal to that of the enclosure. It was also important to ensure that the final temperature of the calorimeter was equal to its initial temperature, since otherwise a part of the energy would have been expended in raising the temperature of the calorimeter and its contents.

At the close of the experiment, this condition being fulfilled, the current was switched off and the valve was closed. The quantity of liquid evaporated was deduced by weighing the calorimeter before and after the experiment.

## § 4. SOURCES OF ERROR

Two possible sources of error might be pointed out: (i) an unbalanced water-equivalent correction, and (ii) want of uniformity of temperature throughout the calorimeter.

As regards (i), calculation shows that if the final temperature of the calorimeter differed by as much as 0·1° C. from the initial temperature the error would be of the order of $\frac{1}{3}$ per cent. There was no difficulty in bringing the final temperature to within 0·1° C. of the initial value.

As regards (ii), the thick metal walls and the tubing of the calorimeter helped to equalise the temperature by conduction. Furthermore, the heating of the bottom layer of liquid would tend to set up convection currents.

In constructing a new calorimeter of this type it might be advantageous to incorporate an electromagnetically operated stirrer such as that described in the authors' paper on "The specific volumes of some gaseous refrigerants."*

## § 5. RESULTS

The liquids studied, with the results obtained, are set out below. Samples obtainable commercially were studied, as these were the fluids of interest in refrigeration.

*Ethyl chloride.* This is a liquid which has already been used to a moderate extent in refrigerating plants. Its thermal properties were carefully studied by Jenkin and Shorthose† and it was therefore a suitable liquid to check the accuracy of the apparatus. The liquid was purchased in 2 lb. tins from a firm of manufacturing chemists. It was analysed for the moisture content, which was 0·4 per cent. in the liquid and 0·6 per cent. in the vapour. Four experiments were carried out, one with the liquid as purchased, and three with a re-distilled sample. They showed no

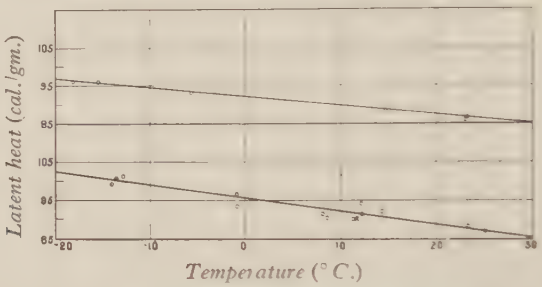



Fig. 2. Latent heat of evaporation of pentane (upper graph) and sulphur dioxide (lower graph). Redistilled material ×.

difference outside the limits of experimental error, the mean values being 92·3 cal./gm. at 14·5° C. and 90·5 at 23·0 C. against Jenkin and Shorthose's values of 92·2 and 89·3 at the corresponding temperatures.

*Methyl chloride.* As with ethyl chloride, measurements were taken on this material merely as a check on the apparatus. The material was used as purchased, its moisture content being of the order of 0·05 per cent. The result found at 10·8° C. was 93·3 cal./gm. as against Shorthose's value of 94·2.

*Sulphur dioxide.* This material was widely used for large plants at one period in the history of refrigeration, but nowadays its field of application is restricted to plant of the domestic type. The results obtained are set out in table 1 and are shown in figure 2.

Over the range investigated the results are sufficiently well represented by a straight line, as shown in figure 2, its equation being $L = 95{\cdot}5 - 0{\cdot}34\,t$, where $L$ is the latent heat in cal./gm. and $t$ is the centigrade temperature. All the above results refer to sulphur dioxide as taken from the cylinders in which it is sold

* See p. 115.

† Food Investigation Board, *Special Report*, No. 14.

commercially. One experiment at 11·7° C. was carried out with sulphur dioxide which had been distilled over sulphuric acid. It is marked in figure 2, and clearly gives a result similar to those from the raw product.

Table 1. Latent heat of sulphur dioxide.

| Temperature (° C.) | Latent heat (cal./gm.) |
|---|---|
| − 14·0 | 99·1 |
| − 13·6 | 100·6 |
| − 12·8 | 101·0 |
| − 0·9 | 96·5 |
| − 0·8 | 93·4 |
| + 8·1 | 91·2 |
| + 8·6 | 90·2 |
| + 11·2 | 90·0 |
| + 12·1 | 94·0 |
| + 12·2 | 91·3 |
| + 14·3 | 92·1 |
| + 23·2 | 88·2 |
| + 25·0 | 87·3 |

*Pentane*. There are two forms of $C_5H_{12}$, the iso- and the normal form, which have boiling-points of about 36·2° and 27·9° C. respectively. It might be mentioned that the values recorded in the literature for the boiling-points are rather variable. The material used was that obtainable commercially. It was fractionated and was found to contain fractions boiling at temperatures from 32·5° to 35·0° C. with no marked preponderance of any particular fraction. The density at 4° C. was 0·6453 gm./ml.

The results are shown in figure 2, from which it appears that the straight line $L - 92{\cdot}2 - 0{\cdot}238t$ represents the latent heat of this material between − 20° and + 30° C.

*Dichloroethylene*. This material is only just beginning to be used at all extensively in refrigeration. Chemically, there are three substances which can be included under this name, and we can trace no previous determinations of the latent heat of any of them. The substance purchased appears to have been a complex mixture, for after two previous fractionations it showed the following composition:

6 per cent. distilled below 49·0° C.
22 ,, ,, between 49° and 54° C.
8 ,, ,, between 54° and 55° C.
13 ,, ,, between 55° and 56° C.
16 ,, ,, between 56° and 57° C.
22 ,, ,, between 57° and 58° C.
11 ,, ,, between 58° and 58·6° C.
2 ,, ,, above 58·6° C.

The density of the fraction boiling between 49° and 54° C. was 1·266 and that of the sixth fraction (57–58° C.) was 1·278, both taken at 22·5° C.

It has a very high specific volume, and would only be suitable for a turbo-compressor installation designed for cooling large quantities of air through a moderate temperature range. Only four experiments were carried out with the sample that was obtainable, the results being as set out in the first two columns of table 2.

These are best represented by a straight line, giving the smoothed values shown in the third and fourth columns.

Table 2. Latent heat of dichloroethylene.

| Observed values | | Smoothed values | |
|---|---|---|---|
| Temperature (° C.) | Latent heat (cal./gm.) | Temperature (° C.) | Latent heat (cal./gm.) |
| 11·2 | 79·1 | 10 | 79·7 |
| 12·0 | 80·1 | 15 | 79·2 |
| 22·7 | 78·8 | 20 | 78·7 |
| 24·2 | 78·0 | 25 | 78·2 |
| | | 30 | 77·7 |

§6. ACKNOWLEDGMENTS

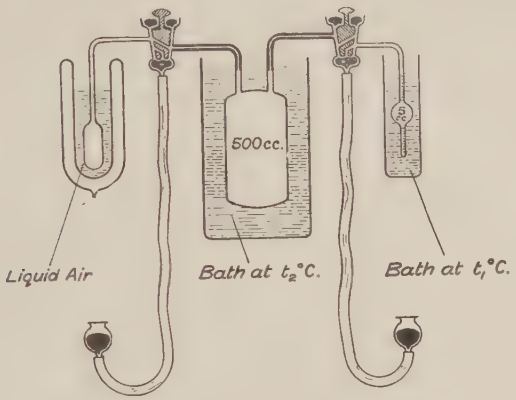
We are indebted to the Engineering Committee of the Food Investigation Board for permission to publish this account of the work, and to the Superintendent of the Physics Department for his interest and for providing facilities to carry out the work. Mr A. R. Challoner, Senior Observer, assisted in the construction of the apparatus and with the observations, and we wish to express our appreciation of his help in the investigation.

## DISCUSSION ON THE PRECEDING TWO PAPERS

Mr T. I. Jones. The authors have successfully applied an idea in the development of which I had a small share about eight years ago. The principle underlying the method was first conceived by Professor A. O. Rankine and was utilized by myself, preliminarily to a measurement of the viscosity of the vapour of carbon bisulphide, for investigating the relation existing between the pressure, volume and temperature of that vapour in the region of pressure and temperature prevailing during the viscosity-measurements. Some years later I found that such a method fulfilled a long-felt want, and I therefore avail myself of this opportunity of making it better known.

The original method differed from the one described in the paper in that the volume of liquid distilled was measured in a calibrated tube, and the suction pump

was replaced by a Dewar flask containing liquid air. The latter provision made possible an indefinite number of repetitions of the experiment with the same small original quantity of liquid. Mercury-seal taps lubricated with graphite were used,




Vapour-density apparatus.

and a pellet of mercury was introduced into the tubes to safeguard the taps on the excess-pressure sides. The method possesses the singular advantage that it can measure the density of a saturated vapour.

Mr A. G. Tarrant said that the papers might give rise to the impression that while sulphur dioxide and ammonia were noxious to human beings, dichloroethylene was not. He personally had worked with trichloroethylene and considered it more dangerous than the first-mentioned gases, in consequence of its being odourless.

Mr J. H. Coste (communicated): As regards the paper on specific volumes, I do not feel quite certain as to the authors' statement that the residual air after the first exhaustion does not affect the quantity of $SO_2$ required to fill the vessel at any given pressure. At 0° and 760 mm. the density of $SO_2$ is 0·0029266, which is only about 1 in 1000 above that corresponding to its molecular weight on the assumption that it is as perfect a gas as oxygen. As the exhaustion was fairly good the error, if any, on twelve fillings would be small. I presume that the re-establishment of temperature-equilibrium was adopted as the criterion of equilibrium between the contents of the end vessels, but if determinations of the pressure of the vapour could have been made these would have added to the value of this very useful piece of work.

The work on latent heats is useful, not only in connection with the immediate technical problems of the authors, but as furnishing data for reference on a subject which has not been much investigated. From this point of view it was to be regretted that liquids of technical quality only were investigated, but the reason for this is obvious. A very similar method has recently been used for a variety of organic liquids, mostly of higher boiling-point than those which are used for

refrigerants, by J. H. Mathews and P. R. Fehlandt*, who determined the time for evaporation of (usually) 10 gm. by means of a constant current flowing through a heating-coil.

As to the substances themselves, it may not be inappropriate to point out that methyl chloride, owing to its poisonous properties and the lack of a distinctive and unpleasant smell by which leaks may be detected, has been found to be unsuitable for multiple refrigerators with one central compression unit in large residential buildings. Many cases of illness have occurred in the United States from this cause.

There are three isomeric pentanes normal, iso- and tetra-methyl methane, boiling respectively at about 37°, 30°, and 9·5° C., but the technical product probably contains only the normal and the iso- isomers as the authors seem to have found. There are three dichloroethylenes, an unsymmetrical one $CH_2 : CCl_2$, which is not of technical interest, and two symmetrical ones, which are formed together by the action of chlorine on acetylene. These comprise the *cis* form, boiling at 60·25° C. $\begin{matrix} HCCl \\ \vdots \\ HCCl \end{matrix}$, and the *trans*, boiling at 48·35° C. $\begin{matrix} HCCl \\ \vdots \\ ClCH \end{matrix}$†. The dielectric constants differ in a remarkable degree. The authors were evidently working with the usual technical mixture of these, which is used largely as a solvent and to some extent in surgery.

Dr A. Ferguson. These very interesting papers emphasise the need for the accurate determination of the physical properties of carefully purified liquids and of the variation of these properties over a wide range of temperature. The dependable data are surprisingly meagre, and one reason for this is not far to seek. The preparation and purification of the liquids are matters for the chemist, and the determination of their physical properties is more immediately the business of the physicist. If the work is carried out by a chemist, we hear complaints concerning the accuracy of his thermometry and the like; while the physicist who adventures on the task is likely to be reproached for his lack of care in testing the purity of his specimens.

Professor Sydney Young has shown how the problem may be successfully attacked, and his paper is the classic of its kind. Physicists and physical chemists are to be congratulated in that the authors are turning their attention to these problems. I trust that they will be able to extend the scope of their work both in respect of temperature-range and of the nature of the substances studied it would be difficult to overestimate its value. The investigation would involve the measurement of the orthobaric densities of liquids and of their saturated vapours and the corresponding vapour-pressures over the whole range of their normal existence, culminating in the determination of their critical constants. This task has been accomplished by Young for some thirty pure organic substances; the results are to be found in the paper already mentioned‡.

* *J. Amer. Chem. Soc.* 53, 3212–3217 (1931).

† J. Errera and M. Lepingle, *Bull. Acad. Roy. de Belg.*, Classe de Science, 5, 150 (1925).

‡ *Proc. R. Dub. Soc.* June (1910).

Such results can be accumulated but slowly, and it would be very helpful if the authors could be persuaded to obtain for us reliable values of the critical constants, especially of the critical density and temperature.

Appeal may then be made to a known law connecting reduced density and reduced temperature which, for unassociated substances at least, will give us approximate values of the orthobaric densities of the liquid and of its vapour at any required temperature. Some time ago I deduced such a relation from certain surface-tension formulae. Thus, it is well known that, over the whole temperature-range of existence of an unassociated liquid, we have for the relation between surface tension $\gamma$ and density

$$\gamma = C(\rho_e - \rho_v)^4,$$

where $C$ is a constant and $\rho_e$, $\rho_v$ are the orthobaric values of the liquid and vapour densities respectively. $C, \rho_e, \rho_v$

If we eliminate $\gamma$ between this and the power law for the variation of $\gamma$ with temperature

$$\gamma = A(1 - m)^n,$$

where $A$ is a constant, $m$ is the reduced temperature and $n$ may be taken as equal to 1·2, we arrive at an equation for $\rho_e - \rho_v$. But the law of rectilinear diameters gives $A, m, n$

$$\rho_e + \rho_v = a - bm,$$

where $a$, $b$ are constants. If now we determine the various constants involved from the considerations that at the absolute critical temperature $\theta_c$ we have $\rho_e = \rho_v = \rho_c$, where $\rho_c$ is the critical density, and that when the temperature $\theta = 0$, $\rho_v$ is negligible and $\rho_e = 4\rho_c$, we easily find by addition that $a, b$ $\theta_c, \rho_c, \theta$

$$\rho_e = 2\rho_c[(1 - m)^{0\cdot3} + 1 - 0\cdot5\, m],$$

and subtraction gives a similar equation for $\rho_v$. If we regard this as a relation between the reduced values of the orthobaric density and the temperature, we see that it contains no constant characteristic of the substance in question.

Young's experimental data serve to test the equation. Writing it as

$$(\rho/2\rho_c) - (1 - 0\cdot5m) = (1 - m)^{0\cdot3},$$

we see that a plot of the logarithm of the left-hand side against the logarithm of $(1 - m)$ should yield a straight line through the origin, of slope equal to 0·3.

The substances so far investigated give very definitely straight lines. The slope of each line is very near to 0·3. No line goes through the origin. This indicates that the equation should be amended to

$$\rho = 2\rho_c[A(1 - m)^{0\cdot3} + (1 - 0\cdot5m)],$$

where $A$ is now a constant characteristic of the substance. $A$ does not, however, vary very greatly from substance to substance and may to a first approximation be taken as equal to 0·9. The formula has been tested for a number of substances, and represents the march of density with temperature very accurately. In one or $A$

two instances it is necessary to take account of the deviation of the index from 0·3 if the highest accuracy is necessary. Some examples of the equation follow:

$$\text{Benzene: } \rho = 2\rho_c\,[0{\cdot}8790\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$$
$$\text{Methyl formate: } \rho = 2\rho_c\,[0{\cdot}9192\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$$
$$\text{Chlorobenzene: } \rho = 2\rho_c\,[0{\cdot}8995\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m]$$
$$\text{Pentane: } \rho = 2\rho_c\,[0{\cdot}8760\,(1-m)^{0{\cdot}29} + 1 - 0{\cdot}5\,m]$$
$$\text{Fluorobenzene: } \rho = 2\rho_c\,[0{\cdot}8831\,(1-m)^{0{\cdot}3} + 1 - 0{\cdot}5\,m].$$

A comparison of observed and calculated values is given for benzene in the table.

Table: Values for benzene

| Temperature (° C.) | $\rho$ (observed) | $\rho$ (calculated) | Temperature (° C.) | $\rho$ (observed) | $\rho$ (calculated) |
|---|---|---|---|---|---|
| 0 | 0·8992 | 0·9001 | 160 | 0·7185 | 0·7185 |
| 20 | 0·8792 | 0·8790 | 180 | 0·6904 | 0·6906 |
| 40 | 0·8585 | 0·8576 | 200 | 0·6602 | 0·6605 |
| 60 | 0·8371 | 0·8357 | 220 | 0·6266 | 0·6255 |
| 80 | 0·8153 | 0·8145 | 240 | 0·5878 | 0·5851 |
| 100 | 0·7926 | 0·7927 | 260 | 0·5391 | 0·5328 |
| 120 | 0·7690 | 0·7692 | 280 | 0·4615 | 0·4514 |
| 140 | 0·7442 | 0·7440 | (Critical) 288·5 | 0·3045 | 0·3045 |

The figures for benzene show the closeness of the agreement between the observed and calculated values. It will be seen that there is a slight deviation in the neighbourhood of the critical point, which may possibly be connected with corresponding deviations from the law of rectilinear diameters. As may be expected, the equation subsumes a number of short-range and other empirical formulae which have from time to time been put forward. These and other allied matters I hope to be privileged to discuss before the Society in the future.

It may be *à propos* to point out here a mode of attack on the problem of the relation between latent heat and other physical quantities which has not been explored as fully as it might be. If we assume a characteristic equation $f(p, v, t) = 0$ which gives curves of the van der Waals type, and if then we draw across any given isothermal, below the critical point, the horizontal line which represents the course of change from the saturated vapour to the liquid phase, the well-known rule for the equality of areas gives

$$p\,(v_2 - v_1) = \int_{v_1}^{v_2} p\,dv,$$

which may be integrated by substitution of the value of $p$ given by the characteristic equation. We have, further, two other equations, since $v_1$ and $v_2$ are abscissae of points on the isothermal under discussion, and these three equations, together with the Clapeyron equation, form a basis for the discussion of the relation of $L$ to other physical quantities. The matter is not free from pitfalls*.

* See Kuenen, *Die Zustandsgleichung der Gase u. Flüssigkeiten*, p. 90.

AUTHORS' reply. We wish to thank all the contributors to the discussion for their helpful suggestions. We are particularly indebted to Mr Coste and Dr Ferguson for their remarks, which will be of value in further work on the subject. The present stage of the enquiry is concerned with the development of methods of measurement, and the apparatus has been tried out on fluids of technical interest.

The differences in form between the apparatus shown in Mr Jones's diagram and the one used by us is an example of how proportions and details must be modified to suit the general magnitude of the quantity studied, and to fit in with the chemical characteristics of the materials present. We had to deal with fluids of high vapour-pressures, so the reservoirs and stopcocks had to withstand considerable internal pressures. The apparatus of Mathews and Fehlandt referred to by Mr Coste is intended for liquids with boiling-points above normal room-temperatures. It resembles that devised by Ramsey and Marshall in 1896*. It could not be readily adapted to deal with the liquids we were studying.

Mr Tarrant raises a point that industry will do well to remember: of two equally poisonous substances, the more dangerous is the one which gives no warning of its presence.

Dr Ferguson's equations for saturation densities are most interesting. He has apparently found a law of considerable generality, and we shall look forward to a detailed account of it in due course.

His remarks on the use to which Maxwell's "equality of area" law might be put are also of interest. The same idea had occurred to us in connexion with other work, but we had hoped to be able to use it in the converse manner. Very accurate empirical vapour-pressure equations are available, and it seemed that it might be possible to proceed from one of them to deduce the general nature of the class of characteristic equations which could lead to this particular vapour-pressure law. If this could be done, we should be a considerable step forward on the road to determining a really reliable equation of state. Unfortunately the mathematical difficulties are very great, and pressure of other work has caused us to abandon this particular enquiry.

* *Phil. Mag.* **41**, 38 (1896).

# THE BASIC LAW OF THE WET-AND-DRY-BULB HYGROMETER AT TEMPERATURES FROM 40° TO 100° C.

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*ABSTRACT.* The relation between relative humidity and wet-and-dry-bulb readings, at dry-bulb temperatures between 40° and 100° C., has been studied experimentally. The relative humidity was measured both by absorbing and weighing the moisture in a known volume of air, and by the dew-point method. The results are finally expressed by means of a skeleton table for the wet-and-dry-bulb hygrometer in the temperature range referred to.

## § 1. INTRODUCTION

Practical experience with various hygrometers has shown that the wet-and-dry-bulb type is particularly well adapted for certain classes of measurements, such as, for example, the determination of humidity in a region of high temperatures where observations have to be taken from some point outside the heated zone. The wet-and-dry-bulb is also a convenient form of hygrometer for adaptation to the control of humidity, and consequently we find that the majority of the appliances developed for the operation of kilns for the drying of timber are based upon the wet-and-dry-bulb principle.

Whilst much attention has been given to the development of the mechanical side of this form of hygrometer, comparatively little has been done to establish the validity of the basic formulae by means of which the observations are reduced to relative humidities.

The best known tables for use with the ventilated type of psychrometer are those published by the Prussian Meteorological Institute*, which extend to a maximum temperature of 40° C. only for the dry bulb.

Of tables proceeding to 100° C., only two are known to us. One, prepared by the staff of Messrs A. B. Cleworth, is published by Francis and Taylor, and the other, in the form of a chart, was prepared by Tiemann and is to be found in his book on the *Kiln Drying of Timber*†. The Cleworth book of tables gives no information as to its mode of preparation, nor any indication as to the degree of ventilation of the wet bulb to which the tables apply. Tiemann's chart is stated to have been constructed by extrapolation from determinations at lower temperatures

* *Aspirations Psychrometer-Tafeln.* (Vieweg und Sohn, Braunschweig.)

† Published by the Lippincott Co.

and to have been checked by experiment at a few points, but no details of such tests are available.

It was therefore deemed advisable to carry out a systematic study of the fully ventilated instrument in the temperature range 40–100° C., so that tables could be prepared depending directly on the results of experiment. For the purpose of this investigation, the actual humidity was measured by two independent methods, the dew-point method and the gravimetric method.

## § 2. APPARATUS

The apparatus was arranged to deliver a stream of air initially heated, and then mixed with steam to raise its humidity to an appropriate value, into a vessel containing the wet-and-dry-bulb thermometers and the dew-point apparatus together with means for aspiring samples of the air at the same point for gravimetric determinations.

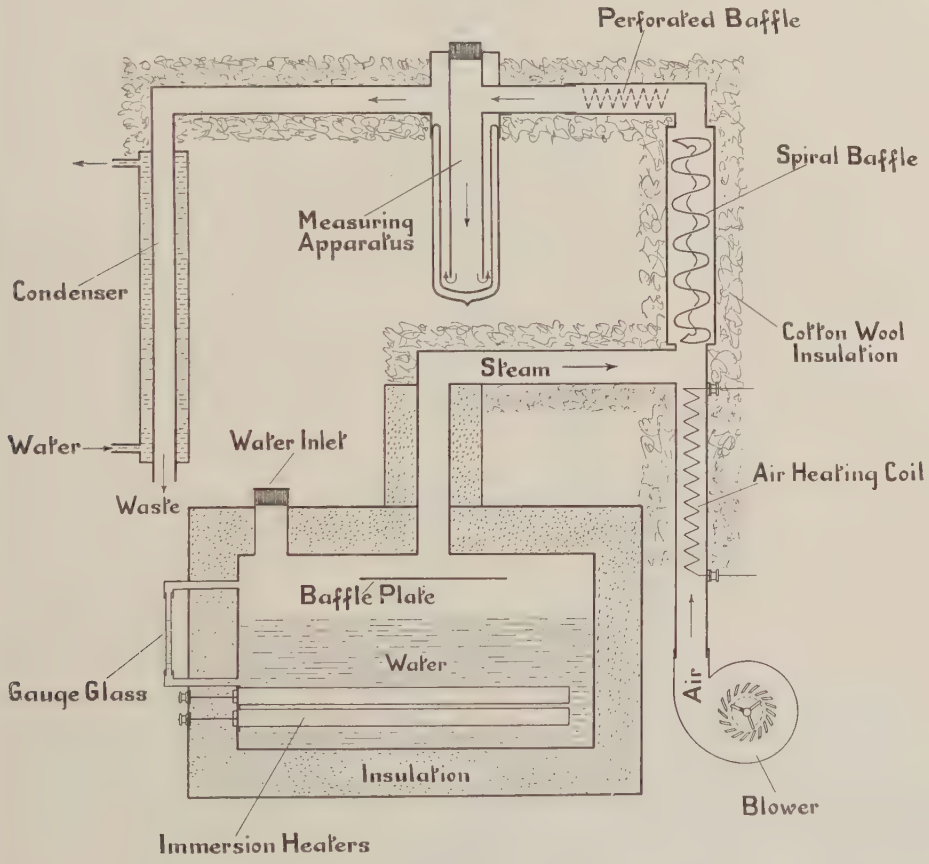


Fig. 1. Diagram of apparatus.

A section of the apparatus is shown in figure 1, the details of the measuring instruments being omitted. The boiler had a capacity of 30 gallons; it was lagged

with magnesia and was provided with four immersion heaters rated at 7 kW. in all, the rate of production of steam being controlled by the energy input. Since the heat losses from the tank were small, the delivery of steam was under accurate control through the electrical input. Air from the room was taken in by the blower, of the Sturtevant type, after which it was heated on its way to the junction with

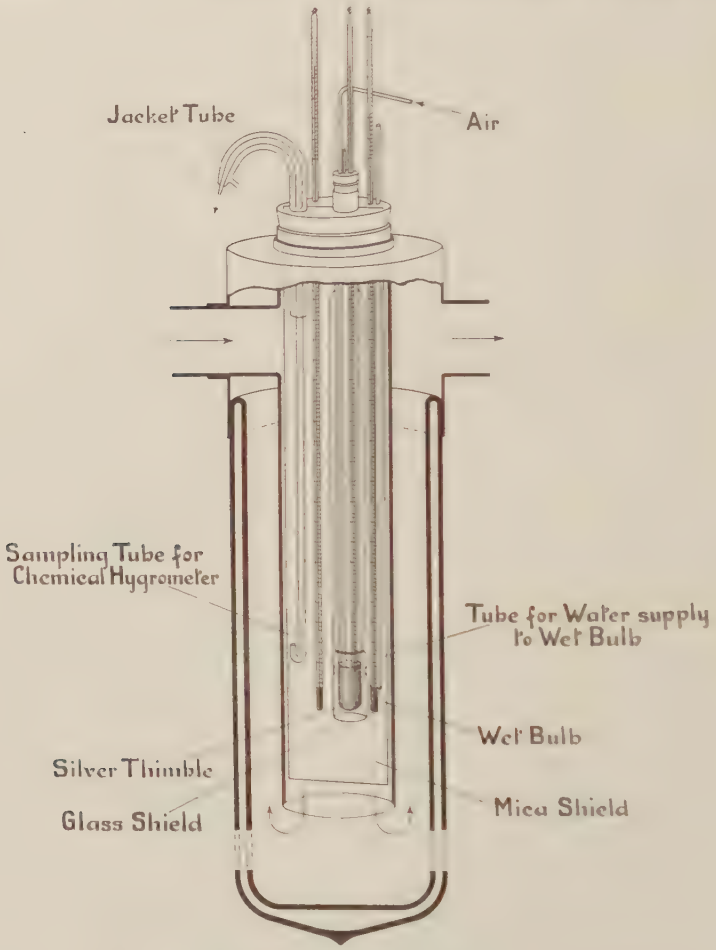


Fig. 2. Measuring apparatus.

the steam supply. This heating was essential, since two variables had to be controlled separately, namely the dry-bulb temperature and the relative humidity. Care had to be taken to ensure thorough mixing of the stream, since experience showed that patchiness was liable to occur, and it should also be mentioned that all pipes were thoroughly lagged with cotton-wool.

The air stream next passed into the measuring apparatus shown in detail in figure 2. To obtain thermal insulation, and yet permit observation of a dew-point thimble, it was found convenient to use a vacuum flask about 40 cm. deep and

11 cm. in internal diameter; centrally within it was a glass tube 6 cm. in diameter and divided down the middle by a thin strip of mica sheet. The tube carrying the air stream to the measuring apparatus entered this glass tube through a brass fitting on the vacuum vessel. The delivery of the air was down the central tube from this point, the air passing up around the outside of the glass so as to jacket the latter and prevent heat losses from the contents; after which it was led away to waste as shown in figure 1.

The glass tube is the central point to which the rest of the apparatus is auxiliary. It contained the wet-and-dry-bulb thermometers under study, as well as a dew-point thimble and a sampling tube for withdrawing air to an absorption hygrometer. Since the dew-point instrument withdraws moisture from the air, whilst the wet-bulb thermometer is liable to raise its humidity, these two instruments were placed on opposite sides of the mica sheet.

The wet-bulb thermometer, at these temperatures, dries rather rapidly, and it was found necessary to provide a continuous slow feed to the muslin jacket. It should be mentioned that tests were made, when the apparatus was set up, to ensure that the air speed past the wet bulb exceeded 3 m./sec., it having been found that beyond this speed the rate of flow of the air is without sensible effect on the wet-bulb depression.

It is not usual to operate a dew-point thimble in a rapid stream of air, and observations were found to be greatly facilitated by fixing a small glass shield, pierced with holes, around the silver thimble. The liquid used in the tube was ether at lower temperatures, but at higher temperatures this evaporated too quickly and a mixture of ether and alcohol or alcohol and water was found convenient. Cooling was effected by the usual method of bubbling air through the liquid in the silver thimble. All the thermometers were calibrated for fixed immersion, and their height was such that the top of the mercury column projected out of the glass vessels.

Observations on the wet-and-dry-bulb instrument and on the dew-point are of necessity discontinuous, whilst the absorption method is not so. Thus a small part of the air stream entering the measuring vessel was led off to the absorption tubes for the whole duration of each experiment, passing along the tube seen in the figure, through three absorption tubes, and thence to a large aspirator of known volume. That part of the tube leading to the absorption tubes which was outside the vacuum vessel was jacketed with hot air from the vacuum vessel, so as to ensure that no condensation, leading to loss of water, took place before the absorption tubes were reached. The latter were in the form of U-tubes and were charged with pumice soaked with concentrated sulphuric acid, prepared by heating the pumice to bright redness and dropping it into the acid.

The aspirator, which ensured a steady flow of air through the absorption apparatus, was of metal and had a capacity of about 20 litres. It was operated with water, which was permissible since the air had at this stage passed the absorption tubes, and water vapour would not diffuse back against the air stream. In any case, such a possible source of error would have been revealed, since in each case

care was taken to note that the third of the absorption tubes did not change appreciably in weight.

### § 3. OBSERVATIONS

Before commencing an experiment, the warm air was allowed to flow for some time before the steam was turned on, since otherwise the cold surfaces of the glass vessels tended to deposit moisture when the mixed air was admitted. After the steam flow had been switched on conditions were allowed to become steady, and after this preliminary period the aspirator tap was opened and observations on the dew-point and on the wet-and-dry-bulb thermometers began. In most experiments the time for the aspirator to empty was of the order of half an hour, and during this period about 10 observations of the wet-and-dry-bulb instruments, and about 45 or 50 of the dew-point, were taken, these being shared between two observers. When the water meniscus in the aspirator reached a definite mark in a glass tube projecting from its base the flow was stopped, the absorption tubes were plugged up and the pressure and temperature in the aspirator were noted. The absorption tubes were weighed in the usual manner after being allowed to cool.

A typical experiment will serve to show the order of the quantities involved and the corrections applied.

10 observations of dry bulb: Mean = $85{\cdot}7_5$° C., Max. = 85·8, Min. = 85·4° C.
10 observations of wet bulb: Mean = $60{\cdot}3_6$° C., Max. = 60·6, Min. = 60·2° C.
62 observations of dew-point: Mean = $58{\cdot}2_4$° C., Max. = 58·6, Min. = 57·7° C.

The variations in the dry and wet bulbs were parallel to each other, owing to a slow drift in the room temperature. The gain in weight of the absorption tubes was 3·314 gm., including 0·008 in the third tube — a value rather above the average. At the conclusion of the experiment the temperature of the air in the aspirator was 14·7° C. (corrected) and the pressure there was 16 cm. of water below atmospheric, the barometer standing at 755·7 mm. of mercury.

After correction for emergent stem and calibration errors, the various temperatures become:

Dry bulb $85{\cdot}6_8$° C.
Wet bulb $60{\cdot}2_6$° C.
Dew-point $58{\cdot}2_0$° C.

To calculate the relative humidity from the dew-point observation, we have: saturation vapour pressure at $58{\cdot}2_0$° C. = 141·3 mm., and at $85{\cdot}6_8$° C. = 445·8 mm., whence the relative humidity is $\frac{141{\cdot}3}{445{\cdot}8} = 31{\cdot}7$ per cent. Throughout, the vapour pressure of water as a function of temperature has been taken from the *Wärmetabellen* published by the German Reichsanstalt.

From the gravimetric observations we have the known volume of the aspirator as 20,225 cm.$^3$, determined on several occasions by weighing the water required to fill it. Thus the volume of air drawn through the absorption tubes is 20,225 cm.$^3$,

after re-saturation and at a total pressure of $(755{\cdot}7 - 160/13{\cdot}6)$ mm. of mercury and a temperature of 14·7° C. Thus the actual volume at the dry-bulb temperature (85·7° C.) and at barometric pressure is

$$20{,}225 \times \frac{273{\cdot}1 + 85{\cdot}7}{273{\cdot}1 + 14{\cdot}7} \times \frac{(756 - 160/13{\cdot}6 - 13)}{756 - 141},$$

since the partial pressure of the water vapour when the air is in the aspirator is saturation pressure at 14·7° C. (i.e. 13 mm. mercury) and the partial pressure of the water vapour at the measuring instruments is that corresponding to the dew-point (i.e. 141 mm. mercury).

This reduces to 29,970 cm.³ of dry (and therefore, by Dalton's law, of moist) air, at the dry-bulb temperature of 85·7° C. and a barometric pressure of 756 mm. of mercury. This gave up 3·314 gm. of water, so that initially the moisture content in gm./m.³ must have been 110·6.

Now the saturation moisture content in gm. per cu. metre at $85{\cdot}6_8$° C. is $356{\cdot}_0$ from determination at this Laboratory (see the accompanying paper), so that the relative humidity from the gravimetric method is

$$\frac{110{\cdot}_6}{356{\cdot}_0} \text{ or } 31{\cdot}1 \text{ per cent.}$$

Thus the two methods, in this particular case, agree to 0·6 per cent. relative humidity.

## § 4. ANALYSIS OF RESULTS

The results of all the experiments carried out are shown in table 1.

Table 1.

| Temperature of | | Relative humidity | | |
|---|---|---|---|---|
| Dry bulb | Wet bulb | By dew-point | By gravimetric method | Mean |
| 100·6 | 75·0 | 35·3 | 35·6 | 35·4 |
| 100·4 | 82·7 | 50·0 | 51·6 | 50·8 |
| 100·0 | 64·6 | 22·2 | 22·2 | 22·2 |
| 99·8 | 60·2 | 17·6 | 16·7 | 17·2 |
| 99·7 | 70·1 | 29·5 | 29·7 | 29·6 |
| 97·4 | 82·0 | 54·2 | — | 54·2 |
| 96·2 | 78·2 | 48·5 | 49·6 | 49·0 |
| 96·0 | 60·2 | 20·9 | 20·0 | 20·4 |
| 95·5 | 74·4 | 41·8 | 43·4 | 42·6 |
| 95·4 | 80·7 | 56·2 | 59·5 | 57·8 |
| 95·0 | 64·6 | 27·1 | 26·8 | 27·0 |
| 95·0 | 81·0 | 57·2 | 59·8 | 58·5 |
| 94·6 | 70·0 | 35·5 | 34·3 | 34·9 |
| 93·8 | 76·3 | 49·3 | 50·0 | 49·6 |
| 92·4 | 70·5 | 38·7 | 32·9 | 35·8 |
| 90·2 | 64·8 | 32·7 | 33·5 | 33·1 |
| 90·0 | 70·3 | 42·8 | 41·4 | 42·1 |
| 89·8 | 59·2 | 24·1 | — | 24·1 |
| 89·7 | 74·1 | 52·4 | 53·1 | 52·8 |
| 88·5 | 71·1 | 48·3 | 48·0 | 48·2 |
| 86·2 | 43·0 | 8·8 | 9·0 | 8·9 |

Table 1 (*contd.*).

| Temperature of | | Relative humidity | | |
|---|---|---|---|---|
| Dry bulb | Wet bulb | By dew-point | By gravimetric method | Mean |
| 86·1 | 74·0 | 60·7 | 61·2 | 61·0 |
| 85·7 | 70·0 | 50·8 | 50·8 | 50·8 |
| 85·7 | 60·3 | 31·7 | 31·1 | 31·4 |
| 85·6 | 56·5 | 25·3 | — | 25·3 |
| 85·0 | 70·2 | 52·4 | 51·9 | 52·2 |
| 84·0 | 51·7 | 20·7 | 19·9 | 20·3 |
| 84·0 | 56·0 | 27·5 | 25·0 | 26·2 |
| 84·0 | 64·3 | 42·1 | 41·1 | 41·6 |
| 82·1 | 68·0 | 54·6 | 54·0 | 54·3 |
| 80·8 | 67·3 | 56·3 | 57·4 | 56·8 |
| 80·4 | 59·9 | 37·8 | 37·9 | 37·8 |
| 80·4 | 56·0 | 30·2 | 32·7 | 31·4 |
| 79·2 | 51·2 | 24·0 | 24·1 | 24·0 |
| 78·2 | 69·1 | 65·8 | — | 65·8 |
| 76·8 | 66·9 | 65·1 | 63·7 | 64·4 |
| 76·3 | 54·6 | 35·7 | 33·8 | 34·8 |
| 76·1 | 60·0 | 48·3 | 46·4 | 47·4 |
| 76·0 | 46·4 | 20·5 | 19·9 | 20·2 |
| 74·7 | 49·4 | 27·7 | 29·1 | 28·4 |
| 71·0 | 60·0 | 60·0 | 58·3 | 59·2 |
| 70·8 | 55·7 | 47·7 | 46·8 | 47·2 |
| 70·3 | 61·9 | 68·8 | 65·8 | 67·3 |
| 69·8 | 46·1 | 28·0 | 27·5 | 27·8 |
| 69·6 | 49·5 | 34·9 | 33·1 | 34·0 |
| 69·5 | 41·0 | 19·1 | 18·6 | 18·8 |
| 67·8 | 57·7 | 62·8 | 59·7 | 61·2 |
| 65·4 | 41·5 | 24·9 | 23·7 | 24·3 |
| 65·3 | 55·9 | 62·6 | 61·5 | 62·0 |
| 64·8 | 42·2 | 26·5 | 25·5 | 26·0 |
| 64·8 | 48·9 | 42·6 | 42·0 | 42·3 |
| 64·7 | 42·8 | 28·2 | 28·1 | 28·2 |
| 63·4 | 35·7 | 16·5 | 16·3 | 16·4 |
| 60·2 | 46·0 | 44·5 | 44·1 | 44·3 |
| 60·1 | 39·6 | 27·6 | 27·5 | 27·6 |
| 60·1 | 31·8 | 12·4 | 12·8 | 12·6 |
| 60·0 | 49·4 | 56·3 | 55·8 | 56·0 |
| 59·8 | 53·5 | 72·2 | 71·6 | 71·9 |
| 59·3 | 35·9 | 21·0 | 22·0 | 21·5 |
| 59·2 | 38·9 | 28·2 | 29·8 | 29·0 |
| 54·8 | 41·0 | 43·9 | 43·3 | 43·6 |
| 54·6 | 44·6 | 55·3 | — | 55·3 |
| 54·6 | 43·6 | 54·2 | 53·6 | 53·9 |
| 54·4 | 48·6 | 72·7 | 67·6 | 70·2 |
| 54·2 | 34·6 | 25·9 | 28·3 | 27·1 |
| 50·9 | 44·4 | 71·0 | 69·2 | 70·1 |
| 50·4 | 29·8 | 21·0 | 20·6 | 20·8 |
| 50·3 | 44·4 | 72·4 | 67·6 | 70·0 |
| 50·2 | 39·8 | 51·7 | 50·0 | 50·8 |
| 48·8 | 34·5 | 36·8 | 36·3 | 36·6 |
| 45·1 | 24·1 | 14·8 | — | 14·8 |
| 44·5 | 34·8 | 52·0 | 51·6 | 51·8 |
| 44·4 | 31·0 | 35·0 | 35·6 | 35·3 |
| 44·0 | 39·7 | 77·5 | — | 77·5 |
| 40·4 | 35·2 | 70·2 | 70·2 | 70·2 |
| 40·3 | 30·6 | 47·3 | 47·5 | 47·4 |

As a first step in the reduction of these data to a system, the values of the relative humidity were plotted against the depression of the wet bulb for approximately constant dry-bulb temperature. Such a curve is shown in figure 3, where it will be seen, for example, that a change in the dry-bulb temperature of about 34·6° C. (i.e. from top to 3rd curve) changes the relative humidity by 17 per cent. at a depression of 20° C., and by 18 per cent. at a depression of 30° C. Such curves, 14 in number, were used as a basis to correct each observation to the relative humidity that would have been shown had the dry bulb been at the nearest of the round values 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100° C. without alteration of the wet-bulb depression. It is clear both from the above example and from the

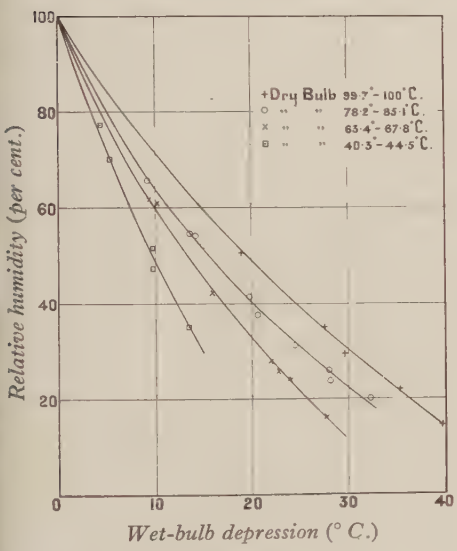


Fig. 3.

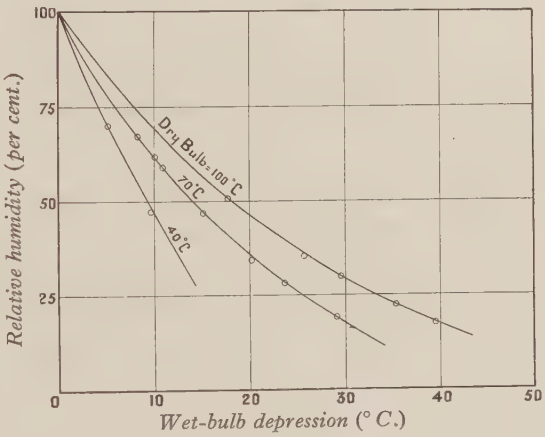


Fig. 4.

well-defined nature of the curve in figure 3 that this alteration will in all cases be small, since no wet-bulb reading had to be altered by more than 2·5° C. Thus, the first six values of relative humidity in table 1 were changed from 35·4, 50·8, 22·2, 17·2, 29·6 and 54·2 to 35·1, 50·6, 22·2, 17·4, 29·7 and 53·1 per cent. respectively.

These adjusted values were next plotted in exactly the same manner as before, and gave points lying much more closely on smooth curves. An example is shown in figure 4. From curves drawn through these points, values were read off at depressions of 5, 10, 15, 20, 25, 30, 35, 40, 45° C., for dry-bulb readings of 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100° C., and gave table 2 below. Points which were doubtful owing to extrapolation are enclosed in brackets.

At this stage of the reduction, it is known that the values in any one vertical column give a smooth curve when plotted against wet-bulb depression, but it is also necessary to ensure that the numbers in a horizontal row give smoothed values against dry-bulb temperature (which would correspond to a slight shift, or a slight distortion, as the case may be, in the curves of which figure 4 is an example).

The result of plotting the numbers in table 2 on this basis is shown in figure 5, to which, however, additional points have been added below 40° C., dry bulb. The latter points are taken from the *Aspirations Psychrometer-Tafeln* referred to in the introduction. The smoothed curves drawn in figure 5 do not depart appreciably

Table 2.

| Wet-bulb depression (° C.) | Dry-bulb temperature (° C.) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 |
| 5 | 71·6 | 73·1 | 73·8 | 74·8 | 76·9 | 77·8 | 78·5 | 80·7 | 80·8 | 81·5 | 82·0 | 82·3 | 83·6 |
| 10 | 46·8 | 50·2 | 52·8 | 55·3 | 58·2 | 59·7 | 61·2 | 64·2 | 64·5 | 65·7 | 66·5 | 67·5 | 68·9 |
| 15 | — | 31·5 | 35·8 | 39·7 | 43·0 | 45·0 | 47·3 | 50·0 | 51·0 | 52·5 | 53·7 | 55·2 | 56·5 |
| 20 | — | 16·2 | 21·7 | 26·6 | 30·0 | 32·6 | 35·5 | 37·9 | 39·6 | 41·4 | 42·9 | 44·8 | 45·9 |
| 25 | — | (0·0) | 10·0 | 15·5 | 18·7 | 22·2 | 25·6 | 27·8 | 30·1 | 32·0 | 33·6 | 35·8 | 36·4 |
| 30 | — | — | (0·0) | (5·9) | 8·6 | 13·2 | 17·1 | 19·2 | 22·0 | 24·2 | 25·7 | 28·0 | 29·2 |
| 35 | — | — | — | — | — | (5·9) | — | 12·0 | — | 17·4 | 18·9 | 21·3 | 22·6 |
| 40 | — | — | — | — | — | — | — | — | — | 11·5 | — | 15·5 | 17·0 |
| 45 | — | — | — | — | — | — | — | — | — | (6·2) | — | — | — |

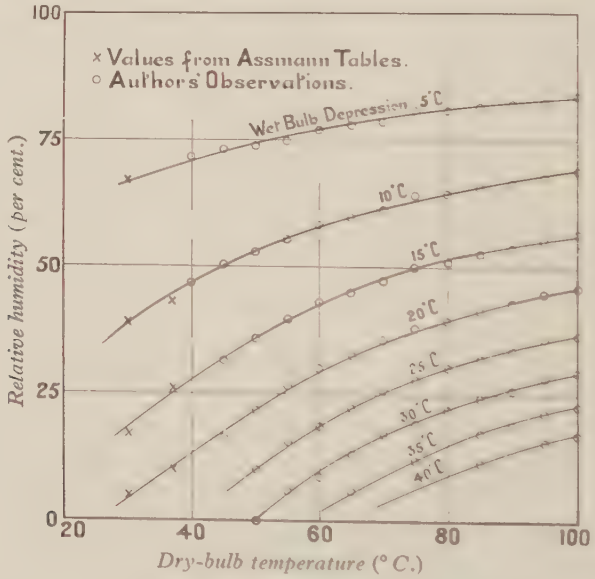


Fig. 5.

from the points of table 2, and it has been assumed that values taken from the smoothed curves will still give curves approximately smooth when plotted against wet-bulb depression. Points taken from the smoothed curves of figure 5 have the advantage that they join smoothly on to the well-established tables prepared for use at lower temperatures, and they are in consequence tabulated, as our final smoothed values, in table 3.

Although the values of relative humidity are given to tenths of 1 per cent., it is not to be assumed that they are reliable to better than 1 per cent. The additional figure is given to assist those who wish to plot the results or to prepare more detailed tables, using table 3, which is only a skeleton, as the basis. In any application of the results the values, whether interpolated or not, should be rounded off to the nearest 1 per cent.

Table 3. Final smoothed values*.

| Wet-bulb depression (° C.) | Dry-bulb temperature (° C.) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 5 | 66·4 | 70·7 | 74·3 | 77·0 | 79·3 | 81·0 | 82·4 | 83·3 |
| 10 | 38·6 | 46·9 | 52·9 | 57·6 | 61·4 | 64·5 | 66·9 | 68·7 |
| 15 | 18·5 | 27·5 | 35·7 | 42·6 | 47·7 | 51·5 | 53·9 | 55·9 |
| 20 | (4·2) | 13·3 | 21·9 | 29·6 | 35·0 | 39·8 | 43·2 | 45·8 |
| 25 | — | — | 10·0 | 18·7 | 25·2 | 30·1 | 33·8 | 36·5 |
| 30 | — | — | — | 9·9 | 16·9 | 22·2 | 26·0 | 29·2 |
| 35 | — | — | — | (2·0) | 9·0 | 15·0 | 19·4 | 22·8 |
| 40 | — | — | — | — | (3·6) | (9·1) | 13·5 | 17·2 |
| 45 | — | — | — | — | — | (4·0) | (8·7) | — |

* Values not within the experimental range are enclosed in brackets.

§ 5. ACKNOWLEDGMENTS

We have to thank the Superintendent of the Physics Department for the facilities necessary to carry out this work. Mr A. Snow assisted in the design of the apparatus and with the initial observations, whilst Mr M. J. Hickman, Junior Observer, assisted with the greater number and suggested useful modifications in the apparatus.

## DISCUSSION

Dr J. H. Brinkworth remarked that the authors had found that when the speed of the air past the wet bulb exceeds 3 m./sec. the depression is independent of the rate of flow. He enquired what was the effect of variation in the rate of flow below this speed? He thanked the authors for emphasising the importance of the use of dummy tubes in the direct weighing experiments.

Mr R. S. Whipple said that hygrometric measurements were being made at higher temperatures than was formerly the case and he knew from experience that the data provided by the authors were urgently needed.

Major W. S. Tucker referred to the problem of hygrometric measurements on high-flying airplanes, where very low temperatures had to be dealt with. If the authors could extend their work to low-temperature conditions the results would be valuable in connexion with another investigation at present in hand.

Mr A. Blackie. In the past one has been apt to regard results obtained from wet-and-dry-bulb hygrometers with a considerable degree of doubt. These experiments put the subject on a much firmer basis.

Authors' reply. (In reply to Dr Brinkworth): With the particular form of apparatus used it would be difficult to determine the quantitative relationship between the air speed and the wet-bulb depression. The wet bulb would need to be placed in a wind channel to obtain uniform air-flow past it. In the experiments a high rate of air-flow was maintained and by varying the conditions it was ascertained that small increases or decreases in the air-velocities used had no measurable effect on the results obtained, so it was presumed that the velocities were adequate for the purpose we had in view. (In reply to Major Tucker): It is proposed to extend the investigation to low temperatures, and particularly to the range which is of interest in refrigeration work.

# THE WATER-CONTENT OF SATURATED AIR AT TEMPERATURES UP TO 100° C.

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*ABSTRACT.* The water-content of air which is just saturated has been measured at temperatures from 29° to 94° C. by absorbing the moisture and weighing it. To obtain saturation, air was bubbled through water in a thermostatically controlled bath.

In some experiments the air was initially below saturation, whilst in others it contained more moisture than would be necessary for saturation at the bath temperature. It was verified that the results agreed in the two cases.

## § 1. INTRODUCTION

The quantity of water vapour which can be retained by air at any temperature $t$ from $-40°$ to $+40°$ C. has been tabulated in the *Aspirations Psychrometer-Tafeln* prepared by the Prussian Meteorological Institute. No reference is given as to the origin of these data, but they are possibly a recalculation of Regnault's results which are given in their original form in Kaye and Laby's tables*. Regnault's data are the only comprehensive measurements of which we are aware.

Assuming Dalton's law, which may be expressed roughly in the form that "two gases may be mixed without affecting each other" (so that if the two occupy a volume $v$, and either alone in the same volume would have the pressure $p_1$ or $p_2$, then the total pressure is exactly $p_1 + p_2$), it would follow that the mass of water vapour which just saturates 1 cm.$^3$ of air at $t°$ C. is the same as that which would just occupy 1 cm.$^3$ at the same temperature in the absence of air. This latter quantity is not easy to measure directly, but is easily calculated from Clapeyron's equation $dp/dt = t^{-1} L/(v_2 - v_1)$, where $p$ is the vapour pressure of water, $t$ the absolute temperature, $v_2$ the reciprocal of the quantity referred to and $v_1$ is the volume of 1 gm. of liquid water at the same temperature, whilst $L$ is the latent heat in mechanical units.

Values produced in this way up to and beyond 100° C. are given by the German Reichsanstalt in their *Wärmetabellen*. It seems desirable, however, in view of the fact that these are not experimental results, and that they involve differentiating the vapour-pressure curve, as well as of the fact that Dalton's law cannot be assumed without proof, to carry out direct determinations, at any rate up to 100° C.

* Sixth edition, p. 41.

## § 2. EXPERIMENTAL ARRANGEMENTS

The method adopted was to saturate a current of air and then to absorb the moisture from a measured volume in sulphuric acid, and to determine its quantity by weighing. Very careful precautions must be taken to avoid even the slightest cooling of the air after it has been saturated, since this would involve a deposition of moisture and consequently a low result.

The apparatus employed is shown in section in figure 1. In one method of experiment, the air was first heated and then mixed with steam, so that the resulting mixture was at a temperature above that for the intended experiment and contained water in excess of the amount that would ultimately saturate it in the experiment. It then entered the apparatus (at the left-hand side of figure 1) and passed through

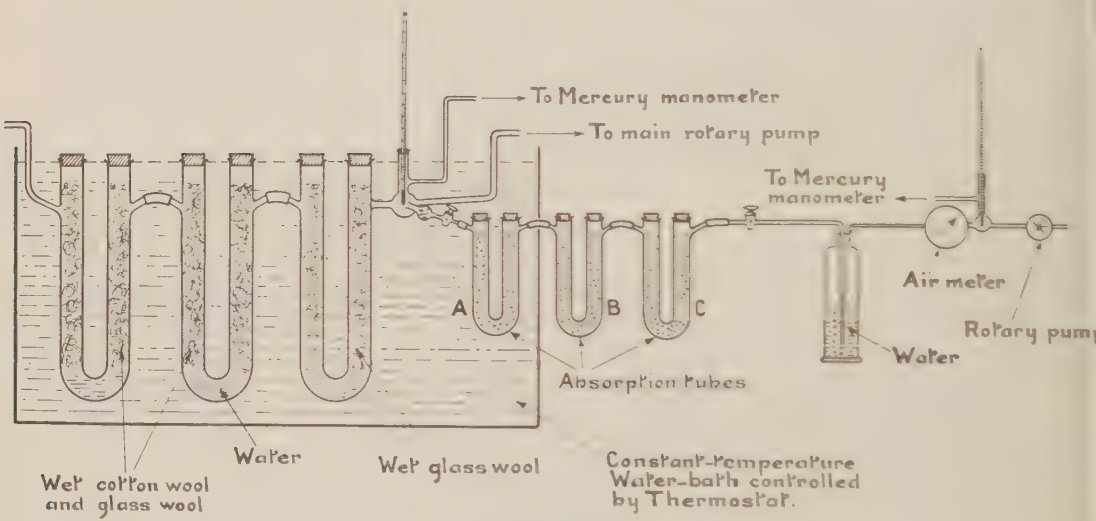


Fig. 1. General diagrammatic view of apparatus.

three U-tubes which contained wet glass wool and were maintained at a constant temperature by immersion in a thermostatically controlled water-bath. From these U-tubes a sample of the air was by-passed to the absorption tubes containing the sulphuric acid, the first of these also being in the controlled water-bath, so that the greater part of the contained moisture was removed before any change of temperature could occur.

After leaving the absorption tubes, the air was metered by a gas-meter of the wet type, passing first through wash-bottles which served also to indicate the rate of flow when the apparatus was being adjusted before an experiment. The temperature and pressure at the meter exit were recorded, as well as those immediately before the absorption tubes were entered. The Wright meter employed was calibrated from time to time throughout the work by comparison with a known volume.

The absorption tubes may be mentioned here. They contained broken pumice,

saturated with the sulphuric acid, so as to present a large surface. To impregnate the pumice with acid, the former was raised to a white heat and dropped into the acid. In carrying out an experiment, the air was allowed to run for some time, with the by-pass tap closed, until conditions became steady. The meter was then read, the tap was opened, the pump was started in the by-pass circuit, and absorption was allowed to take place. At the end of the experiment, the absorption tubes were plugged as rapidly as possible and allowed to cool for weighing. It should be noted also that each of the absorption tubes was provided with a dummy of approximately the same dimensions, and weighted with sand. This was desirable owing to the rather small weight of the moisture (as low as 1·5 gm. in one experiment) compared with the total weight measured (possibly 150 gm.) and to the known vagaries of glass in picking up moisture from the air on its surface in quantities sufficient to affect the weight.

A second method was employed which contrasts with the one just described, in that the air initially supplied to the apparatus was drier than that ultimately required, instead of containing excess moisture. As in the other method, the air was passed through the three U-tubes containing wet glass wool, and from these to the absorption tubes. The glass-wool tubes in this case, however, were partly filled with water, through which the air was forced to bubble, their function now being to supply moisture, and not to remove it.

*Results.* The calculation of a typical experiment is set out in full below, in order to show the corrections applied and the methods of reduction employed.

### § 3. SECOND EXPERIMENT OF APRIL 16, 1930

The barometer (corrected) was 762·7 mm. and the reading of the manometer at the outlet of the gas-meter was 18·0 mm., so that the air left the meter at a pressure of 744·7 mm. Its volume at this stage was 1·008 ft.$^3$ as read by the meter, or $1{\cdot}008 \times 0{\cdot}02831$ m.$^3$, account being taken of the calibration error. Now the manometer at the entrance to the absorption tubes read 9·0 mm., so that the pressure at this point was 753·7 mm. These pressures are total pressures, and must in each case be reduced by the saturation vapour-pressure of water at the temperature prevailing at the corresponding position, since the air is saturated at both positions. These temperatures are 70·5° C. and 12·5 C. respectively, the corresponding vapour-pressures being 238·8 mm. and 10·9 mm. Thus the volume of the air when it entered the absorption tubes was

$$\{(273{\cdot}1 + 70{\cdot}5)/(273{\cdot}1 + 12{\cdot}5)\}\{(744{\cdot}7 - 10{\cdot}9)/(753{\cdot}7 - 238{\cdot}8)\} \times 1{\cdot}008 \times 0{\cdot}02831,$$

i.e., $0{\cdot}04893$ m.$^3$.

The differences in weight between the absorption tubes and their dummies were as below:

| | Before experiment | After experiment |
|---|---|---|
| Tube A | 0·755 | 10·564 |
| Tube B | 0·222 | 0·326 |
| Tube C | 0·406 | 0·407 |

These show gains of weight equal to 9·809, 0·104, 0·001, a total of 9·914 gm. Consequently the result of the experiment is to show a moisture content, for saturation, of $9{\cdot}914/0{\cdot}0489_3$ or 202·7 gm./m.$^3$.

It is to be noted that the pressure-measurements which have been used are necessary only for finding the actual volume of the air, at the point where it is saturated and at the temperature under investigation, and not for correcting the saturation moisture-content to a standard pressure. If Dalton's law were absolutely true, the moisture-content would be identical at all pressures from zero upwards, and even though it is not strictly true, the variation over the range of barometric pressures may be neglected. In a few cases experiments were carried out, at approximately the same temperature, by both the methods described above. Typical pairs are shown in table 1.

Table 1. Comparison of methods 1 and 2.

| Temperature (° C.) | Moisture-content (gm./m.$^3$) | |
|---|---|---|
| | Method 1 | Method 2 |
| 60·3 | 131·9 | 132·4 |
| 70·4 | 203·5 | — |
| 70·5 | — | 202·7 |
| 75·0 | 245·3 | — |
| 75·1 | — | 244·1 |
| 85·0 | 343·6 | 343·4 |

Similarly, pairs of observations which differed only in the rate of air-flow were found to show no systematic variation due to this cause.

The results are shown in table 2 and plotted in figures 2 and 3, in which the continuous lines represent the results given in the *Wärmetabellen* of the Reichsanstalt,

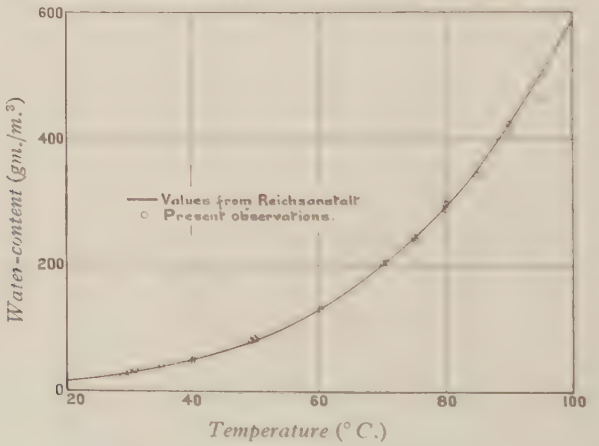


Fig. 2. Moisture-content of saturated air.

and the points are shown individually. If the absolute content be plotted in grams of moisture per cubic metre of saturated air, as in figure 2, the results of the two investigations agree up to about 85° C., after which temperature the present results

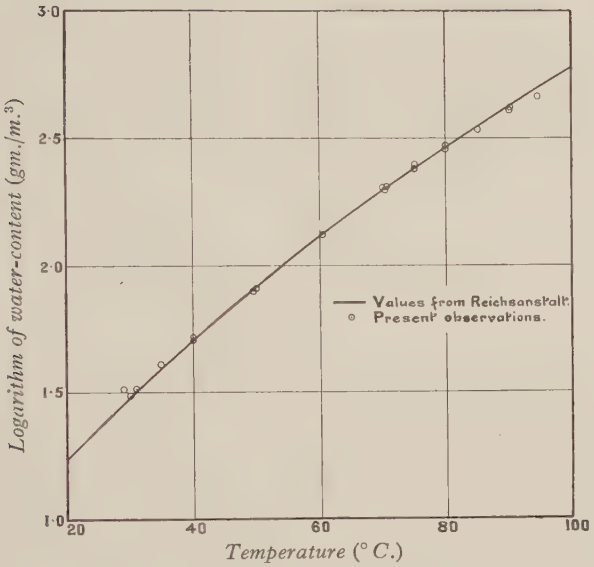


Fig. 3. Moisture-content of saturated air.

Table 2.

| Temperature (° C.) | Moisture-content $Q$ (gm./m.³) | log $Q$ | Weight of moisture collected (gm.) |
|---|---|---|---|
| 29·0 | 32·74 | 1·5151 | 2·46 |
| 30·0 | 31·13 | 1·4932 | 3·07 |
| 31·0 | 32·95 | 1·5179 | 1·50 |
| 35·0 | 41·34 | 1·6164 | 3·12 |
| 40·0 | 52·97 | 1·7240 | 6·55 |
| 40·1 | 51·68 | 1·7133 | 6·59 |
| $40{\cdot}1_5$ | 52·89 | 1·7234 | 2·60 |
| 49·6 | 80·38 | 1·9052 | 4·22 |
| 49·9 | 82·62 | 1·9171 | 4·63 |
| 50·1 | 81·74 | 1·9124 | 11·91 |
| 60·3 | 131·90 | 2·1202 | 16·92 |
| 60·3 | 132·43 | 2·1220 | 11·10 |
| $70{\cdot}0_5$ | 205·28 | 2·3124 | 9·96 |
| 70·4 | 203·49 | 2·3085 | 11·49 |
| 70·5 | 202·74 | 2·3069 | 9·91 |
| $70{\cdot}6_5$ | 206·27 | 2·3144 | 6·59 |
| 75·0 | 245·27 | 2·3896 | 13·72 |
| 75·1 | 244·05 | 2·3875 | 14·02 |
| 80·0 | 289·68 | 2·4619 | 17·36 |
| 80·0 | 298·56 | 2·4750 | 9·89 |
| 85·0 | 343·56 | 2·5360 | 12·72 |
| 85·0 | 343·42 | 2·5358 | 9·95 |
| 90·1 | 408·40 | 2·6111 | 14·50 |
| 90·2 | 422·82 | 2·6262 | 16·28 |
| 94·4 | 464·17 | 2·6667 | 25·90 |

fall on a lower curve than the German ones. On a logarithmic scale, figure 3, in which a given displacement represents the same percentage difference at all temperatures, it appears that the present results cut across the others at an angle. This makes it appear unlikely that the low values at the high temperatures are due to lack of saturation, since if there were such a systematic error, it would exist (and even be larger) at low temperatures.

## § 4. SMOOTHED VALUES

No formula has been found which represents the results very well. The best two are of the forms

$$\log U = A + B/t + C \log t + D\,(t_c - t)^{\frac{1}{2}} + E\,(t_c - t)$$

and

$$p/U = a + bt + ct^2 + dt^3,$$

where $U$ is the saturation moisture-content at the absolute temperature $t$, $t_c$ is the critical temperature of water, and $p$ is the vapour-pressure of water at temperature $t$.

The first of these formulae is of a form used by the Bureau of Standards in their work on ammonia, and has the theoretical advantage that it behaves correctly in the neighbourhood of the critical temperature.

The second formula is a generalization of one used in the Assmann psychrometer tables, viz., $p = Kut$, where $K$ is a constant. The best values resulting from this work will be taken as the mean of the smoothed values derived from the two formulae, the constants in them having been determined by the method of least squares. These adopted values are set out in table 3.

Table 3. Smoothed values adopted to represent the results of the experiments.

| Temperature (° C.) | Moisture-content of saturated air (gm./cm.) |
|---|---|
| 25 | 24·2 |
| 30 | 31·7 |
| 35 | 41·1 |
| 40 | 52·8 |
| 45 | 67·2 |
| 50 | 84·7 |
| 55 | 105·9 |
| 60 | 131·3 |
| 65 | 161·7 |
| 70 | 197·6 |
| 75 | 239·9 |
| 80 | 289·2 |
| 85 | 346·7 |
| 90 | 412·8 |
| 95 | 489·0 |
| 100 | (576·0) |

The accuracy is probably about 2 per cent. throughout, although the same number of decimal figures is retained in every case for convenience when differences are in question.

§ 5. ACKNOWLEDGMENTS

I have to thank Dr Kaye, Superintendent, for facilities for carrying out this work. The apparatus used was adapted and modified from that designed by Dr Griffiths and myself for studying the wet-and-dry-bulb hygrometer at higher temperatures, and I feel deep gratitude to Dr Griffiths for his helpful advice in many matters pertaining to the subject of humidity-measurement. Mr Hickmann assisted in the design and construction of the apparatus, whilst his patient care in taking observations was of the greatest value.

## DISCUSSION

Mr J. H. Coste (communicated): Mr Awbery is probably acquainted with the paper by Vernon and Whitby* on the quantitative humidification of air, in which air and steam were cooled to the desired temperature and then allowed to pass through a long length (about 75 ft.) of glass tubing in the thermostat. Five determinations of the water vapour in air at 25° C. agreed in all cases within 0·5 per cent. with Regnault's value of 22·8 mg./litre, although it is not clear whether they actually represent the water *in* or that *associated with* the volume of dry air which was finally measured. If the latter, their results should be reduced by about 3 per cent. The agreement of the results obtained by the author's methods (1) and (2) seems to remove all doubt as to the close approximation to saturation obtained in the very simple saturator.

Dr A. R. Lee (communicated): In view of the fact that one of the methods employed by the author introduced bubbling, I should like to draw his attention to a paper by Vernon and Whitby† on the quantitative humidification of air. These workers concluded that the most efficient method of obtaining saturation was one similar in principle to that first used by the author. They also showed, however, that it was impossible to obtain saturation merely by bubbling the air through water, and reference is made to other workers who have noted the defects of such a method. Failure is presumably due to the relatively low vapour-pressure within a bubble caused by the concavity of the liquid surface, and also to the condensation caused by the release of pressure when the bubble bursts. The good agreement which the author obtains between his two methods suggests that the wet glass wool is able to compensate for such deficiencies. It seems worth while, however, to call attention to this point, since the errors due to the simple bubbling method are very frequently overlooked.

Mr A. Blackie. I would like to ask whether the bubbling-bottle shown before the air-meter was found, in conjunction with the meter itself, to saturate the air at the temperature in the meter? Personally I have found the greatest difficulty in saturating air by bubbling. I would also draw attention to the really large effects that the condensation film of moisture on glass or rubber tubing has on the moisture

* *Trans. Farad. Soc.* **27**, 121 (1931). † *Ibid.* **27**, 248 (1931).

contained in the air passing through it when there are changes in the original hygrometric state of the air. This should not have troubled the author, however, as he maintained constant conditions for considerable periods.

Author's reply. I thank both Mr Coste and Dr Lee for their remarks on the problem of saturating air. The paper to which they call attention had escaped my notice, and I am glad to learn of its existence. I was well aware that it was practically impossible to saturate air by bubbling it through water, just as it is difficult to obtain saturation in the converse case when air is passed through a spray of water. I suspect that diffusion inside the bubble, or in the space between the droplets, is an important factor, and that subdivision of the bubbles is therefore important, so that lengthening of the time of contact would be valuable. Doubtless the glass wool tends to be of assistance in this way. Mr Blackie's warning as to the change of weight of glass vessels over periods of several hours cannot be too strongly emphasised. I have used the precaution, in various pieces of work, of having a dummy tube of the same size and subjected to the same conditions, and have found it very satisfactory for work of moderate accuracy. The device is by no means new, but its originator is unknown to me.

# SOME PROPERTIES OF THE SOUND EMITTED BY AIRSCREWS

By C. F. B. KEMP, A.R.C.S., B.Sc., D.I.C.

*Received August* 30, 1931. *Read December* 4, 1931

*ABSTRACT.* A condenser-microphone amplifier system has been employed to determine the intensities and directional properties of the first six harmonics in the sound of rotation of an airscrew operating at zero rate of advance and actuated by a silenced engine. The sound-energy associated with the frequencies considered has been found to be 18 watts, the fundamental (first harmonic) being responsible for 50 per cent. of this, while the first three harmonics together contribute 90 per cent. Maximum intensity occurs from 15° to 30° behind the plane of rotation, and in this region the sound-output is particularly steady. Large intensity-fluctuations occur along the axis of the slip-stream. The intensity follows the inverse-square law of distance at points further than 200 ft. from the airscrew centre, a 2·4-power law holding approximately for nearer points. Cathode-ray oscillograms demonstrate the existence of frequencies of the order of 600 ~ which are probably not components of the sound of rotation. The hypotheses of Lynam and Webb, as modified by Paris, are briefly discussed in relation to the results of experiment.

## § 1. INTRODUCTORY

The low-frequency sound generated by a rotating airscrew is known to comprise a harmonic train of frequencies commencing with one equal to the number of blades × airscrew revolutions per sec.[1, 2, 3]; and some experiments made by Fage[4] on model airscrews 3 ft. in diameter indicate that the harmonics become progressively smaller in intensity. The sound is also known to have directional properties, this aspect of the subject being first treated by E. J. Lynam and H. A. Webb[5] and later by M. D. Hart[3]. Recently E. T. Paris[6] has made an experimental study of the intensity-distribution of the fundamental component around a stationary aeroplane and has discovered that the alternative theories of Lynam and Webb may be suitably combined to give a result agreeing well with his observations. An analysis of the sound emitted by aircraft in flight has been attempted by J. Obata and Y. Yosida[7], whose results, so far as airscrew sound is concerned, are discussed at a later stage.

In this paper further full-scale experiments to determine the properties of airscrew sound are described, information being given regarding (i) the rate of intensity-decay at moderate distances from the airscrews; (ii) the directional properties of the first six harmonics in the "sound of rotation"; (iii) the absolute intensities of these components; and (iv) the existence of high-pitched sounds.

Earlier investigators obtained certain results which were not well understood and the bearing of recently acquired knowledge on these is briefly discussed.

## § 2. FACTORS AFFECTING THE CHOICE OF AN EXPERIMENTAL METHOD

Ideally, observations should be made on an airscrew operating under normal flight-conditions by causing an aircraft to maintain a constant height and known rectilinear course with reference to the sound-receiving apparatus. The interpretation of any results so obtained is rendered difficult for the following reasons.

The exhaust sound, which is comparable in intensity with the airscrew sound, has its own appropriate series of harmonics and usually has directional properties of a complicated nature. If the aircraft is one in which the airscrew is driven directly on the crankshaft, the frequencies present in the airscrew sound of rotation must necessarily coincide with certain frequencies in the exhaust sound, so that separation of the two effects is impossible. An aircraft equipped with a geared airscrew is therefore preferable for purposes of experiment. To illustrate the general nature of the aircraft sound in this case, the engine and airscrew finally used by the author will be taken as an example. The engine was a Rolls Royce Condor developing 600 h.p. at 1800 r.p.m.; it had twelve cylinders arranged in two banks of six, each bank having its own exhaust manifold. The two-bladed airscrew was geared down in the ratio 0·477 : 1. The subject of exhaust sound has been treated in considerable detail by M. D. Hart[3] and he has shown that for cylinder-in-line engines the sound consists of a harmonic series of tones having a fundamental $f_E$ given by:

$$f_E = \text{crankshaft r.p.m.} \times \tfrac{1}{60} \times \tfrac{1}{2} \times \text{number of cylinders per bank}$$
$$= 90 \sim \text{for the above engine.}$$

Each component frequency has its own directional effect, the nature of which is dependent upon the relation between the wave-length and the spacing of the exhaust manifold outlets. The frequency of the fundamental airscrew note is $f_A$ where

$$f_A = \text{crankshaft r.p.m.} \times \tfrac{1}{60} \times \text{no of blades} \times \text{gear ratio}$$
$$= 28{\cdot}6 \sim \text{for the above airscrew.}$$

At 1800 r.p.m. the following frequencies are therefore to be expected:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Airscrew | 28·6 | 57·2 | 85·8 | 114·4 | 143·0 | 171·6 | 200·2 etc. |
| Exhaust | | | 90 | | | 180 | etc. |

These two trains of frequencies being non-simply related, continuous oscillograph records of the sound show no wave-form repetitions, this condition being aggravated by the continual change in aspect of the aircraft by reason of which the relative importance of the various exhaust harmonics at the distant recording point also changes. Interpretation of the record could be effected by enharmonic analysis, but in the absence of any rapid method of accomplishing this, the practicability of the idea seems doubtful in view of the great number of analyses required to obtain the relationship between the sound spectra and the aspect of the airscrew.

Selective sound-receivers, tunable to any desired frequency in the sound, would

yield results easier of interpretation, provided that the tuning arrangements were such as to produce a band-pass-filter response, the width of the band being great enough to allow for change in frequency due to the Doppler effect. Some difficulty in interpretation arises, however, in those frequency ranges where the airscrew sound has a frequency component close to one forming part of the exhaust sound (e.g. 85·8 ~ and 90 ~ in the instance quoted above), for, in general, the frequency-separation is less than the frequency-change due to the Doppler effect.

Whatever system of sound-measurement is employed, the time track of the aircraft must be accurately determined, and further, local meteorological conditions must be such that the longitudinal axis of the aircraft coincides with the direction of flight.

For a first study of airscrew sound it was deemed advisable to use an airscrew rotating at a fixed point on the ground, and to neglect, temporarily, any effect due to ground reflection. Expense did not permit of the airscrew being actuated electrically, with the consequent elimination of exhaust noise, and recourse was had to a silenced Rolls Royce Condor engine of the type mentioned above.

## § 3. AIRSCREW EQUIPMENT

The engine and airscrew were mounted on the fuselage of a Berkeley aircraft, the wings, tail plane and rudder having been removed. By placing a heavy trestle of adjustable height under the rear of the fuselage, the plane of rotation of the airscrew could be made vertical, and to prevent the equipment tipping forward, the back was securely lashed to screw pickets in the ground. As a further precaution, all the engine controls were removed from the cockpit to a remote control box, it being thought that some danger existed of the structure ultimately breaking up as a result of running the engine at normal flying speed under these conditions. The fuselage was placed with the two exhaust outlets above a silencing pit, 10 ft. deep, filled with steel shavings and covered with a firm layer of earth at the top. The gases from each exhaust manifold were conducted into the pit through a flexible iron pipe 4 in. wide attached to the normal exhaust outlet. From the pit, the gases were conveyed underground through 50 ft. of iron pipe 6 in. in diameter to a second pit, also filled with steel shavings but having no earth cover, so that escape to atmosphere was possible. Arrangements for cutting out the silencer were made by providing, at the junction of each exhaust outlet with its flexible connexion, an alternative path leading direct to atmosphere. These paths could be opened or closed by spring-controlled baffles, and besides enabling the degree of silencing to be assessed, functioned as a safety device by preventing the accumulation of unburned mixture within the pit when the engine was being started. The performance of the silencing system was found to be improved by wrapping several layers of asbestos blanket round the pipes leading to the pit.

The directional effect of any component in the airscrew sound of rotation may be completely represented by a polar surface with the airscrew boss as origin, the

length of any radius vector being proportional either to the pressure-amplitude or to the intensity in that direction. In the absence of any sound-reflection this surface may, for reasons of symmetry, be regarded as a surface of revolution about the airscrew axis, the generator being the polar curve taken in any plane perpendicular to the plane of rotation of the airscrew and including the axis of the airscrew. With the equipment described, it is therefore sufficient to take observations at points in a horizontal plane through the airscrew axis, if ground-reflection effects at a distant point are to be neglected. The conditions of experiment are simplified if it can be assumed that a polar curve so obtained is symmetrical about the airscrew axis. That this assumption is justifiable was shown by E. T. Paris who, in some early experiments on the fundamental airscrew note, took observations at a fixed point, while an aircraft situated in the middle of the landing ground was turned through 360° in regular angular increments, the airscrew boss being brought always to the same place.

Accordingly, the equipment was installed so that ahead, astern and to port there were no reflecting surfaces other than the ground. To starboard there were no buildings nearer than 150 yds., so that measurements made to port at distances of the order of 100 yds. could be regarded as free from serious errors due to reflection. Even so, the results obtained must be considered strictly as applying to an airscrew operating at zero rate of advance, and at nearly zero altitude, and actuated by an engine which has been silenced as well as circumstances permit.

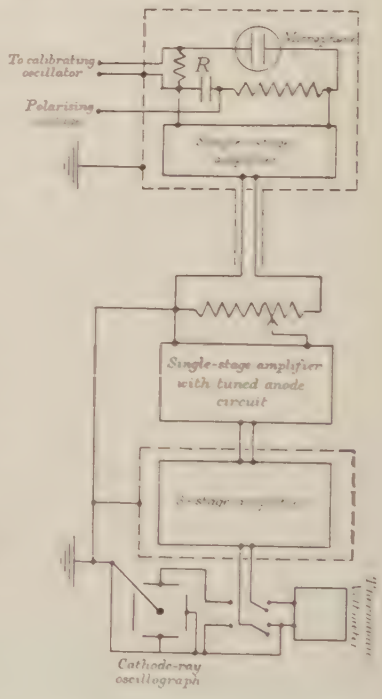


Fig. 1. Schematic of recording apparatus.

## § 4. SOUND-RECORDING APPARATUS

The sound was received by a condenser microphone, the variations in charging-current producing potential changes which were amplified to operate a cathode-ray oscillograph or thermionic voltmeter of wide range, figure 1.

The microphone, a Western Electric model, was calibrated by the resonance tube and Rayleigh-disc method which, for the low frequencies involved, yields the same result as a "free air" calibration[8]. This calibration was performed by the Post Office Engineering Research Department. Up to a frequency of 200 ~ (above which no airscrew sound-measurements were made) the sensitivity was found to be uniformly 2 mV/dyne-cm.$^{-2}$, with reference to the actual r.m.s. pressure on the diaphragm. Amplitude-distortion was assumed to be absent, as it is understood that no condenser microphone of this type has yet been found to suffer from the defect. The first stage of the amplifier was supplied with the

microphone by Standard Telephones & Cables Co. Ltd.; it was housed in a small screened box with the microphone clipped into one side. When observations were being taken, the box was moved around the airscrew and placed upon the ground with the microphone diaphragm horizontal, so that nodal measurements were obtained. This procedure also minimized diffraction effects, and effects due to direct reflection from the ground. It is true that the polar curves were thus taken in a plane not passing through the airscrew axis, but since the points of observation were mostly on a semi-circle of radius 300 ft., the angle included between the horizontal and a line joining the airscrew-centre to the microphone was less than 2°. Isolation of any desired frequency was effected by means of a tuned anode circuit with the resonance curve arranged to show practically uniform response over a small frequency-band determined by the inevitable fluctuations in engine-revolutions and inaccuracies in the revolution meter, while there was only a small response to frequencies about 30 ~ removed from the one considered. To obtain the resonance curves, the microphone was replaced by a condenser of equal capacity (about 0·0003 $\mu$F), a measured alternating current from a precision oscillator was passed through the 2 ohm calibrating resistance $R$, and the output was read on a thermionic voltmeter. The amplifier as a whole was designed so that the overloading of any stage necessarily caused the succeeding one to be overloaded, the effect being indicated by a change in reading of the milliammeter in the plate circuit of the last valve.

For visual observations on pressure amplitude a triple range thermionic voltmeter (0·5 to 120 r.m.s. volts) was used, and for the recording of wave-form a Wood cathode ray oscillograph (the tuning stage of the amplifier was omitted in this case).

## § 5. PRECAUTIONS OBSERVED DURING THE TAKING OF RECORDS

Before the commencement of a series of observations it was ascertained that (i) background noises caused negligible response of the apparatus; and (ii) a light surface wind was blowing. If the atmosphere is still, undesirable refraction effects are introduced by the lapse rate in the stagnant air just above the ground.

No readings were taken when any other aircraft approached the aerodrome.

## § 6. INTENSITY/DISTANCE RELATION

Measurements of pressure-amplitude are most conveniently made at distances of the order of a few hundred feet if the airscrew is operating under the conditions specified. For greater distances it is seldom possible to mark out a semi-circular distribution of observation points without introducing into the sound-field some obstacle which would adversely affect the free passage of the waves; furthermore the pressure-amplitudes are of such a magnitude that, at a few hundred feet, a normal background disturbance produces a negligible effect on the recording instrument. It is imperative, however, that measurements be made at distances great enough for the inverse-square law of intensity to be obeyed, if a knowledge

of the intensity at more distant points is required. An experiment was conducted, therefore, to determine the intensity distance relation at distances from the airscrew between 50 ft. and 500 ft. The determination was made in one direction only, namely 25° behind the plane of rotation, since preliminary experiments had shown that the sound-output was particularly steady and comparatively great in this direction. The following method of taking readings was adopted. A forward observer signalled when the required engine revolutions had been reached and thereafter this speed was maintained for half a minute, a variation of ± 10 r.p.m. being permitted. During this period the output-meter was read at frequent equal intervals and an average value was found. The results are given in table 1, and are consistent with the inverse-square law $I \propto R^{-2}$ when 200 ft. $< R <$ 500 ft. and with $I \propto R^{-2\cdot4}$ approximately when 50 ft. $< R <$ 200 ft.

Table 1. Intensity/distance relation.
(Engine speed = 1700 ± 10 r.p.m.)

| Distance $R$ from airscrew boss (ft.) | Pressure-amplitude × arbitrary constant |
|---|---|
| 490 | 1·0 |
| 400 | 1·25 |
| 300 | 1·55 |
| 200 | 2·3 |
| 100 | 6·0 |
| 50 | 10·85 |

It is improbable that the inverse-square law holds at very great distances, dissipation and refraction effects causing the intensity to decay at a greater rate.

## § 7. INTENSITY AND DISTRIBUTION OF HARMONICS 1–6

On a semi-circle of 300 ft. radius with the airscrew boss as centre, thirteen microphone positions were marked out at intervals of 15°, the first ($\psi = 0°$) being directly ahead of the airscrew and the last ($\psi = 180°$) directly astern. The output-meter readings fell into three classes: (i) constant; (ii) slowly fluctuating between well-defined limits; (iii) as (ii) but with occasional excursions well beyond these limits. In constructing the polar curves of figure 2 these excursions were neglected. The nomenclature adopted identifies the terms "fundamental" and "first harmonic," "octave" and "second harmonic," etc. It appears that (*a*) In general, the higher the order of the harmonic the less the pressure-amplitude, especially for values of $\psi > 75°$. This is in accordance with Fage's model experiments and with information derived from cathode-ray-oscillograph records, § 8. (*b*) Maximum pressure-amplitude occurs between $\psi = 105°$ and $\psi = 120°$, i.e. from 15° to 30° behind the plane of rotation of the airscrew, except in the case of the sixth harmonic which is maximal at $\psi = 150°$. (*c*) Sound-energy is also projected forwards within limits which may be taken approximately as $\psi = 0°$ and $\psi = 60°$. The spur in the polar curve of the first harmonic was also found by Paris, who used a Tucker hot-wire

microphone. At very low frequencies the energy of this part of the sound is small compared with the energy thrown laterally and backward, but becomes of relatively greater importance at the higher frequencies considered, so that the energy in the fourth, fifth and sixth harmonics appears to be distributed fairly equally, though not symmetrically, in front of and behind the plane of rotation.

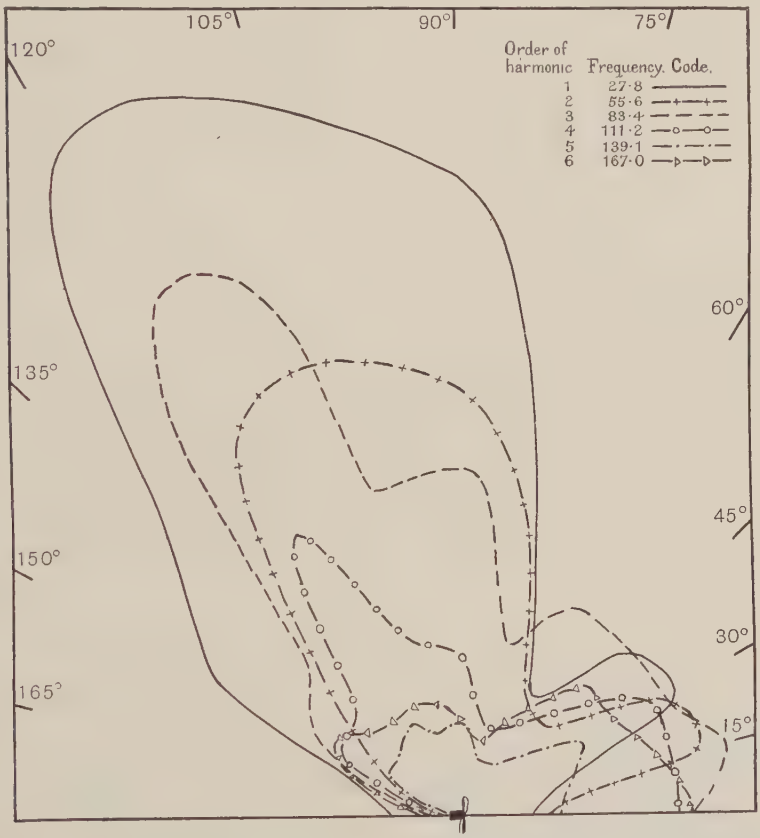


Fig. 2. Polar curves for distribution of airscrew sound. Scale: 1 cm.=0·84 dyne/cm.$^2$ (r.m.s.). (Airscrew pitch at pressure face, 3200 mm.; blade-diameter 4500 mm.; engine speed 1750 r.p.m.)

The lengths of the radii vectores give the actual pressures on the microphone diaphragm, so that the pressures existing in the unimpeded wave-front are half these. An estimate may now be made of the total sound-energy associated with each harmonic, the polar surfaces being generated by the rotation of the two-dimensional polar diagrams of figure 2 about the axis of rotation of the airscrew. (See table 2.)

A study of the pressure-fluctuations observed during each recording period of 30 sec. showed that (1) Along the slip-stream the sound output is exceedingly unsteady, the pressure frequently rising to four or five times the normal value. The effect is quite local, disappearing 15° off the axis. (2) The output is steadiest

between $\psi = 75°$ and $\psi = 165°$. In and near to the direction of maximum output, the amplitude-fluctuation is not more than 3 per cent. while in other directions, within the range stated, the fluctuation is about 5 per cent. for the very low frequencies and not more than 10 per cent. for the higher ones. (3) In any chosen direction the lower the frequency the steadier the output.

Table 2.

Energy in airscrew sound. (Engine speed = 1750 r.p.m.)

| Order of harmonic | Sound-energy (watts) |
|---|---|
| 1 | 8·9 |
| 2 | 3·7 |
| 3 | 3·4 |
| 4 | 1·2 |
| 5 | 0·3 |
| 6 | 0·6 |

## § 8. CATHODE-RAY-OSCILLOGRAPH RECORDS

At the time these experiments were made, the Von Ardenne type of cathode-ray oscillograph[9], suitable for the continuous photography of wave-form, was not available. Dr A. B. Wood's instrument[10], however, yielded useful information in spite of the short time-base, which permitted only about three periods of the fundamental to be included on one plate. Tracings of three typical photographs are shown in figure 3.

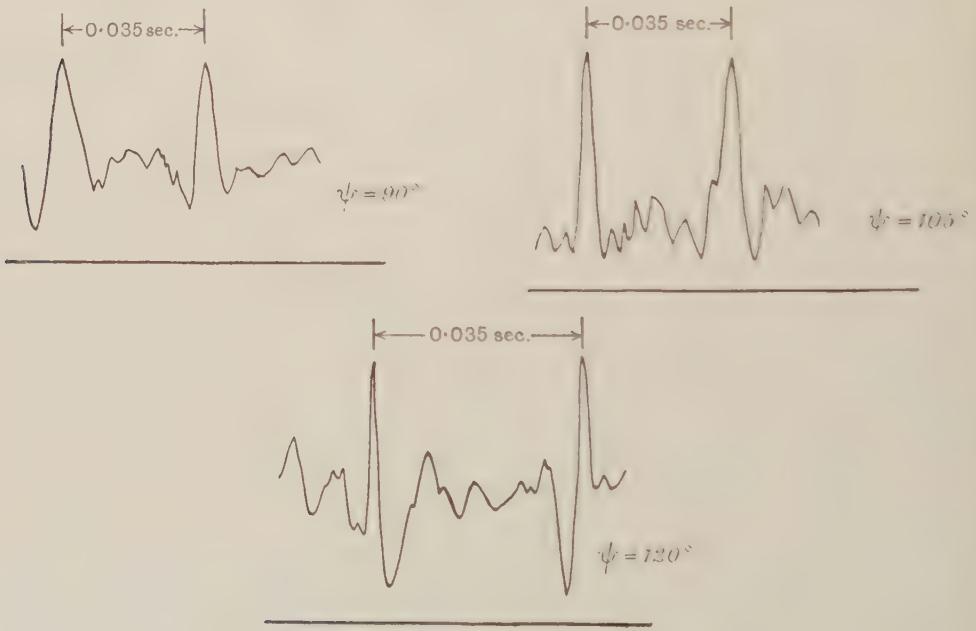


Fig. 3. Tracings of oscillograph records.

Owing to imperfect exhaust-silencing and the presence of engine clatter, wave-form repetitions do not occur with sufficient accuracy for a Fourier analysis to be justified.

The general form of the oscillograms, which were taken near the direction of maximum output, indicate that the sound is of an impulsive character and therefore rich in harmonics; in fact it is more impulsive than at first appears, for the amplification at 30 ~ was only 40 per cent. of that for frequencies within the constant amplification band. On the original plates there was evidence of the existence of a frequency of about 600 ~ superimposed on the main wave-trace. If the harmonics in the sound of rotation decrease in intensity with increasing order, it may reasonably be doubted whether a high frequency which is immediately obvious from inspection forms part of this sound.

A theory as to the production of comparatively high audio-frequency sound by an airscrew has been advanced by H. Glauert. Briefly, this sound (termed the "drag noise") originates in the aperiodic shedding of eddies from all parts of the blades. It is non-directional and of a frequency determined by the tip-speed of the blades. It appears from the oscillograms that the amplitude of this 600 ~ note is small compared with the amplitudes of the low-frequency notes in the sound of rotation.

## § 9. APPLICATION OF THE THEORY OF LYNAM AND WEBB

Lanchester suggested that the sound from an airscrew is due to the movement in circular orbits of pressure-centres of nearly constant magnitude. Lynam and Webb[5] developed this idea, and from a mathematical treatment of the properties of systems of fluctuating acoustical doublets derived expressions for the sound-amplitude in any direction from an airscrew. Their first hypothesis was that the airscrew as a source of sound might be replaced by a ring of sources ahead and a ring of sinks of equal strength at an equal distance astern, both rings being co-axial with the airscrew and parallel to it. In their second hypothesis the sinks were removed to infinity and the sources were placed a very short distance ahead of the airscrew. Both these systems yield zero amplitude along the axis, but whereas the former also gives zero amplitude in the plane of rotation with maxima symmetrically disposed on either side, the latter gives maximum amplitude in the plane of rotation. It was hinted that the truth might be found in a combination of these systems, and this aspect of the subject has recently been considered by E. T. Paris[6].

Assuming the sources of the second system to lie in the airscrew disc and to be $\pi/2$ out of phase with respect to the corresponding sources and sinks of the first system, he obtained the following expression for the sound-amplitude:

$$A \propto J_{pm}(pm\omega a^{-1} R \sin\psi)\{\mu - 2\sin(pm\omega a^{-1} Y \cos\psi)\},$$

where $J_{pm}$ is Bessel's function of order $pm$;

$p$ the order of the harmonic considered;

$m$ the number of blades;

$\omega$ the angular velocity of the blades;

$R$ the radius of the orbits, assumed to be $\frac{3}{4}$ of a blade-length;

$a$ the velocity of sound;

$\psi$ the aspect angle, as defined in figure 2;

$\mu$ the ratio of the strength of the sources of the second system to the strength of the sources and sinks of the first system; and

$Y$ the distance of the sources and sinks of the first system from the plane of the airscrew.

It was shown that this expression yielded a polar curve very similar to the experimentally determined polar curve of amplitudes for the first harmonic when $\mu = 2{\cdot}6$ and $Y = 3 \times$ blade-length. It indicated, however, less output behind the

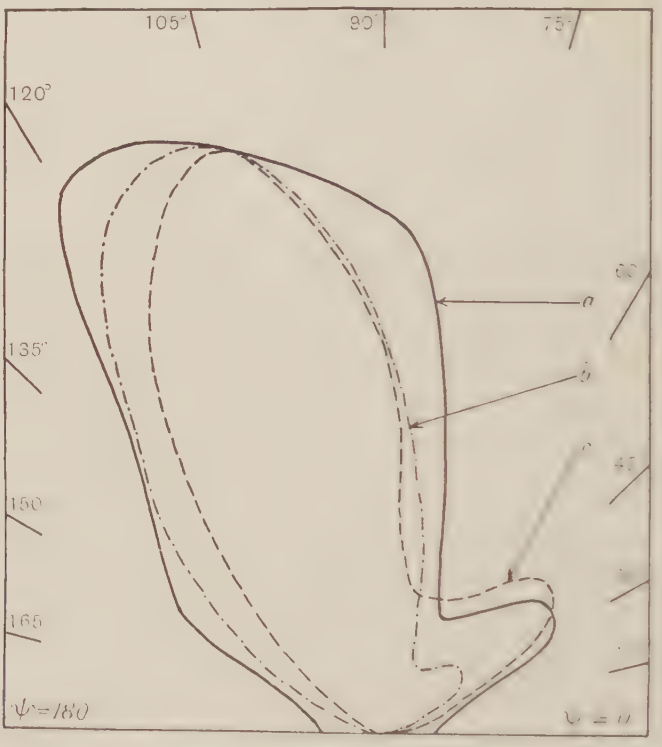


Fig. 4. Pressure-amplitude of first harmonic according to the theory of Lynam and Webb as modified by Paris.
Curve $a$, experimental. Curve $b$, $\mu = 2{\cdot}6$; $Y = 3 \times$ blade-length.
Curve $c$, $\mu = 3$; $Y = 4 \times$ blade-length.

plane of rotation than was found by experiment. So far as the author's measurements of the first harmonic are concerned, the agreement is best for $\mu = 3$ and $Y = 4 \times$ blade-length, the theoretical output behind the plane of rotation again being less than the measured output, figure 4.

Attempts to assign suitable values to $\mu$ and $Y$ for the second and higher harmonics have not been successful. For the second harmonic of a two-bladed airscrew, the value of $J_4(4\omega a^{-1} R \sin\psi)$ for $\psi < 30°$ is inconveniently small compared with the value for $\psi = \pi/2$. The polar curve of figure 2 shows a forward, subsidiary, maximum near $\psi = 15°$, and to preserve the relative amplitudes at $\psi = 15°$ and $\psi = 105°$ (the principal maximum) the value of $\sin(4\omega a^{-1} Y \cos\psi)$ must in practice be large and negative in the former direction, but large and positive at the latter. Since the values of the Bessel function are symmetrical about $\psi = \pi/2$, this implies that the amplitude at $\psi = 75°$ must be much greater than at $\psi = 105°$, which is definitely not so. The same objection applies to values of $pm > 4$, and in any case the appreciable output along the axis at $\psi = 0$ could not be accounted for.

## § 10. TWO NOTES ON THE WORK OF PREVIOUS INVESTIGATORS

(i) *Obata and Yosida*. This work is notable as forming the first published account of experiments to obtain continuous oscillographic records of the sound from aircraft in flight and to correlate the distance and the aspect of the machine with the harmonic analysis. Most of the records refer to an aircraft fitted with a geared airscrew (gear-ratio 29/44) and the analysis should therefore have yielded useful information concerning both airscrew sound and exhaust sound. Unfortunately, the method of analysis adopted is open to serious criticism, for although the numerous lengths of record reproduced in the paper do not exhibit recurring wave-forms, yet a classical Fourier analysis has been applied to certain portions which seem to have been selected in a rather arbitrary manner. In every instance the sound is represented by a single train of harmonics, sometimes commencing with the airscrew fundamental frequency, but more often with the fundamental of the exhaust. The sound spectra given cannot, therefore, be a true indication of the composition of the sound, and without a complete recalculation the effects due to the airscrew cannot be separated from those due to the exhaust. The authors were not concerned with the verification of any theories of sound-emission and admit the weakness in the analytical method. In an approximate manner the analyses show the distribution of energy in the sound spectrum, and in many cases are considered to have a bearing on aural sensations.

(ii) *McKinnon Wood*[1]. In 1919 the Royal Aircraft Establishment made some determinations of the range of audibility of various airscrews actuated electrically near ground level. At the same time observations were made on the frequency of the predominant note in the sound, a pianoforte and pitch pipe being used for comparison purposes. Two installations were employed and different results were obtained from each. When the airscrew was rotated on the spinning tower at a point fixed in relation to the observer, the frequency of the note heard was usually an integral multiple of the frequency given by the (number of blades × r.p.s.). When the airscrew was given a translational motion in a circle 150 ft. in radius by means of the whirling arm, the note heard was not related to the theoretical funda-

mental frequency in a simple manner; further, for a given airscrew, the note perceived was different on different occasions. The directional nature of the sound, not then appreciated, may explain this experience. If the airscrew is advancing in a circular path and an observer is stationed at a fixed point outside this path, the region of maximum intensity in the sound-field associated with the sound of rotation passes the observer twice during each revolution of the arm, i.e. at intervals of 5 sec., the airscrew being assumed to have a forward speed of 90 ft. sec. The directional effect of the intensity is such that the harmonics of low order are relatively loud for about a quarter of this period and comparatively inaudible for the remainder of the period. Under these conditions an estimate of pitch would appear to be very difficult indeed, especially in the presence of a medley of high-pitched sound. Unless the observer were a long way from the path of the airscrew, the loudness would be much greater when the airscrew passed on the near side of the mounting than when it passed on the remote side and this would render the determination of pitch still more difficult. If an airscrew produces a relatively high-pitched sound which is non-directional in character, then this would be heard continuously and would almost certainly receive the prior attention of the observer on account of its continuity and the increased sensitivity of the ear at moderately high audio-frequencies. If the frequency of this sound is not dependent on the angular velocity of the airscrew, the observed note will probably not be recognized as a harmonic of the fundamental sound of rotation.

## § 11. ACKNOWLEDGMENTS

Most of the observations described in this paper have already appeared in various reports submitted to the Aircraft Noise Sub-Committee of the Aeronautical Research Committee. The experiments were conducted under the direction of Dr W. S. Tucker, O.B.E., and the fuselage and airscrew provided by the Director of Scientific Research, Air Ministry. The author is indebted to Dr E. T. Paris for various suggestions and advice tendered during the course of the experiments.

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## DISCUSSION

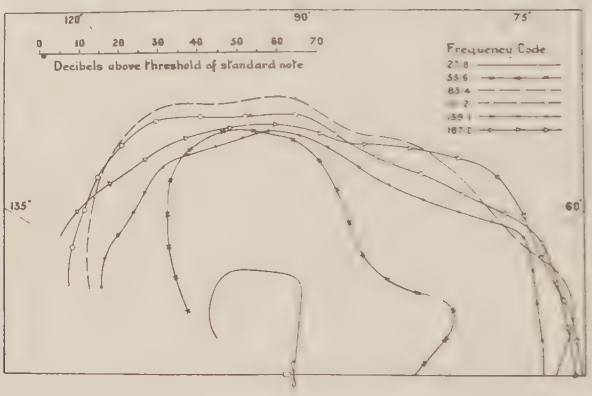
Major W. S. Tucker. The paper describes a difficult piece of team-work done entirely under Mr Kemp's control. The airscrew is a very interesting example of a directive source of sound and I believe this is the first time that the Physical Society has had a paper of this type. I hope that it will be followed up by other contributions of a similar character and will afford some encouragement to others to help in establishing a good theoretical basis for the phenomena which the author has described.

Dr E. T. Paris. The formula discussed in § 9 assumes that $R$ is $\frac{3}{4}$ of a blade-length, and the results are in good agreement with experiment as regards the fundamental tone, but not as regards harmonics. The value $\frac{3}{4}$ was, however, somewhat arbitrarily chosen. Possibly some other value would give more satisfactory results at the higher frequencies.

Dr A. B. Wood. The experimental difficulties encountered in such work are considerable. To an average person standing near a large tethered aeroplane developing its full horse-power, it would seem that the sound-output represents an appreciable proportion of the total horse-power! The author's measurement of 18 watts sound-power from an engine of 600 h.p., i.e. 40 parts of sound-energy per million, shows how unreliable the ear becomes when the sound-intensity passes the "threshold of feeling." The polar-intensity curves shown in figure 2 are very interesting, the striking feature being the backward inclination of the maximum intensity. The impression one obtains from such curves is that the sound-distribution is influenced by the slip-stream. If such is the case, the polar diagram of intensity should change as the rate of revolution of the airscrew is varied. Perhaps the author could supply information on this point.

Dr A. H. Davis. The author has obtained information concerning the fundamental and lower harmonics of airscrew noise which should provide an experimental basis for a reliable theory of airscrew sounds. I was interested to see his curves for the wave-form of the sound emitted by airscrews, for they agree in a general way with the periodic impulsive nature of the wave-form that Mr Lock and I have observed near a model airscrew. On the basis of our experience with this airscrew, which was driven by a comparatively silent electric motor, I suggest that there may be irregularities inherent in the disturbance set up by an airscrew itself, which would explain the irregularities and fluctuations that the author attributed in his case to interference from engine and exhaust noise. We found the wave-form unsteady in detail, although fairly constant in its general characteristics. I would mention that in a preliminary analysis of a noise of a motor-driven airscrew (diameter 3 ft., r.p.m. 1500) Mr Dadson and I found appreciable energy distributed generally and irregularly over the higher-frequency range.

The author mentions that many of his observations have been communicated to the Aircraft Noise Sub-Committee of the Aeronautical Research Committee, but, of course, their value is not confined to the aspects with which that committee is concerned. I would, however, comment that the low-frequency sounds measured are sounds to which the ear is not specially acute. I have made a rough estimate of the loudness that the various components plotted in figure 2 of the paper would have if they existed independently. The results are indicated in the accompanying diagram. It will be seen that the fundamental is the component which is least important to the ear, at any rate at the range 300 ft. at which the author worked. Also the sixth harmonic is more important than the fifth, so that still higher harmonics may, perhaps, be of importance as far as noisiness is concerned.




**Estimated polar curves for distribution of components of airscrew sound expressed in terms of the loudness of each component.**

Dr Paris, to whose as yet unpublished work reference is made, has presented a valuable theoretical equation to express the polar distribution of the fundamental sound around an airscrew. I should be much interested to hear of theoretical calculations which could predict the characteristic wave-form of airscrew noise at any point.

Author's reply. (In reply to Dr Paris): I have not tried the effect of making $R$ different for different harmonics; the value of $\frac{3}{4}$ of a blade-length was selected because the centre of pressure of the blade is situated approximately at this radius. (In reply to Dr Wood): Since the paper was written I have investigated the effect of engine speed on the shape of the polar curve for the fundamental note, and have found no appreciable change to occur. The observations were confined to values of $\psi$ between 90° and 150°, the range of engine speeds being from 1500 r.p.m. to 1800 r.p.m. only. Lack of suitable calibrating apparatus prevented me from working below the former speed, which corresponds to a blade frequency of about 24 ~. (In reply to Dr Davis): The suggestion that there may be inherent irregularities in the sound from an airscrew is particularly interesting. I have found that string-galvanometer records of the sound from a single-engined aircraft in flight usually

show a succession of perfectly repeating wave-forms, indicating steady sound-sources. At times, however, no such perfect repetition is evident—an experience shared by Obata and Yosida*. Meteorological conditions are thought to be responsible for this difference, and if Dr Davis' experiment with the model airscrew was made in the open air, the same disturbing factors may have been operative. I agree that harmonics of higher order than those considered in the paper may be physiologically important. I was more concerned with the purely physical aspect of the problem, and did not proceed to higher frequencies on account of the increasing difficulty of frequency-separation with the simple tuned-anode device described.

* *Loc. cit.*

# SOME THERMOMAGNETIC EFFECTS IN NICKEL AND IRON

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*ABSTRACT.* The paper relates to the production of an e.m.f. in nickel and iron wires by the simultaneous application of a longitudinal magnetizing field and a temperature gradient. This e.m.f. has been measured for various fields and temperatures.

## § 1. INTRODUCTION

THIS paper presents a brief description of the results of some work now in progress. A complete account will be published later, with further results included.

If a magnetic field is applied to a ferromagnetic substance parallel to a temperature gradient, an electromotive force is produced in that direction. The

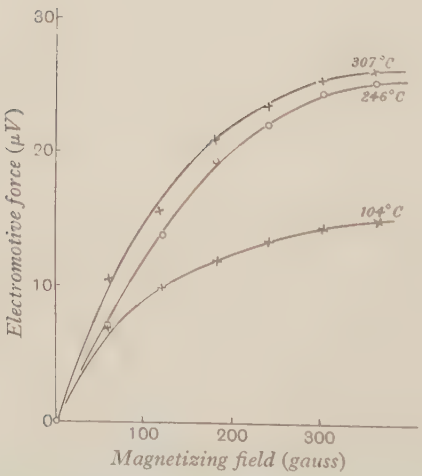


Fig. 1. Results for nickel wire. (Diameter 0·15 mm.; temperature at cold end 14·2° C.; temperature at hot end as indicated against curves.)

Fig. 2. Results for nickel wire. (Diameter 0·15 mm.; temperature of cold end 13·8° C.; magnetizing field as indicated against curves.)

experiments are essentially measurements of this e.m.f., with different fields and temperatures, for wires of nickel and iron. The method used is similar to that employed by Gerlach* who has, however, published results only for nickel.

## § 2. EXPERIMENTAL RESULTS

Figure 1 shows the electromotive force $E$ as a function of the magnetizing field for various temperatures $T$ of the hot end of a nickel wire. Figure 2 shows the value of $E$ as a function of $T$ for various constant values of $H$. These curves will be seen to be almost the same as those published by Gerlach. A very slight hysteresis was also observed, but its magnitude was too small to allow any useful measurements.

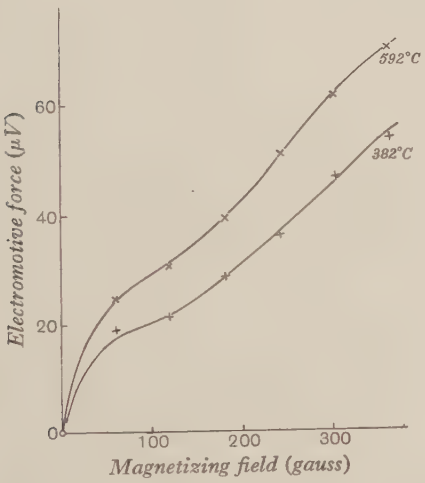


Fig. 3. Results for iron wire. (Diameter 0·57 mm.; temperature of cold end 13·8° C.; temperature of hot end as indicated against curves.)

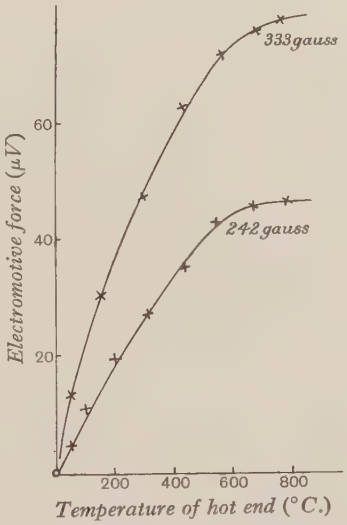


Fig. 4. Results for iron wire. (Diameter 0·57 mm.)

Figures 3 and 4 show $E$ as a function of $H$ for various constant values of $T$, and as a function of $T$ for various constant values of $H$, in the case of an iron wire. The effect in iron is greater than that in nickel, but similar in nature. The magnetic field, however, does not appear to have been strong enough to produce saturation in iron as it did in nickel.

Figure 5 shows the remarkable hysteresis effect in iron at constant temperature with a complete cycle of the magnetizing field. There is an anomalous increase in the value of $E$ just as the value of $H$ is allowed to fall back from its maximum. This anomaly led us at first to suspect that some fault had developed in the apparatus; but it was repeated several times, and no fault could be detected.

* *Proc. Phys. Soc.* **42**, 418 (1930).

Figure 6 duplicates the phenomena Gerlach observed in the neighbourhood of the Curie point. The curves refer to our experiments on nickel, and show that $E$ is

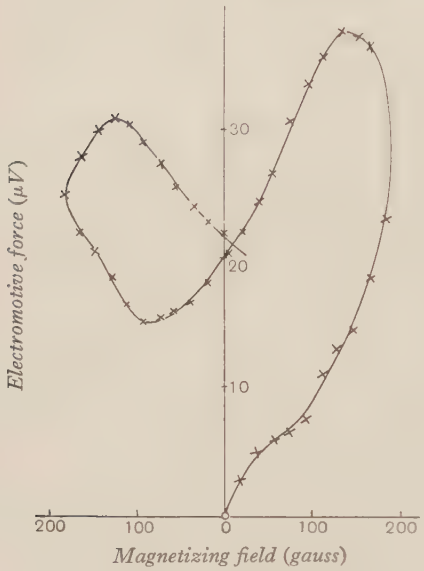


Fig. 5. Hysteresis of electromotive force in iron wire. (Diameter 0·57 mm.; temperature of cold end 13·8° C.; temperature of hot end 210° C.)

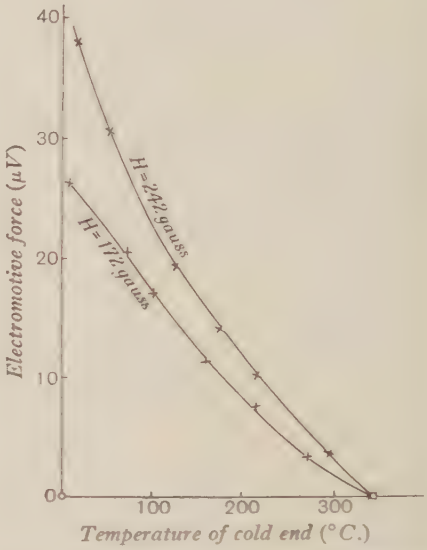


Fig. 6. Results for nickel wire. (Diameter 0·15 mm.; temperature of hot end 390° C.)

independent of $H$ when both ends of the wire are above the Curie-point temperature. There is of course still a finite initial electromotive force when $H$ is zero, even when both ends of the wire are above the Curie-point temperature; but this e.m.f. is unchanged on application of the magnetic field. The e.m.f. was in each case observed to be opposite in direction to the normal Thomson e.m.f. in zero magnetic field.

## DISCUSSION

Dr L. F. Bates. The authors have drawn our attention to some rather interesting thermoelectric effects. The phenomena illustrated by figure 5 of their paper are particularly striking, and they direct attention to one or two aspects of these problems which appear to have escaped notice previously. As their paper represents a preliminary survey we cannot legitimately complain that data concerning the purity of their materials are not given. It is, of course, obvious that some chemical data concerning the iron used will be necessary at a later stage in the investigation. However, even at this stage, a knowledge of the hysteresis curve for the iron specimen would have been of assistance. It is the lack of hysteresis data which leads me to think that the authors have not paid sufficient attention to the work of Ellwood* on the heat changes accompanying the magnetization of iron.

* W. B. Ellwood, *Phys. Rev.* **36**, 1066 (1930).

Ellwood took a series of 104 steel rods, and arranged them in concentric cylinders so that the bunch of steel rods formed an ellipsoid of revolution with a long major axis. Between the steel rods an equal number of copper rods of the same diameter and length were mounted. These iron and copper rods were not in contact, but adjacent copper and iron rods were joined alternately by short pieces of constantan and copper wire, so that a system of 102 thermocouples in series was formed. The whole arrangement was mounted in a special calorimeter inside a solenoid. On changing the magnetic field of the solenoid, heat changes due to magnetization were produced in the iron rods but not in the copper rods. The change in temperature of the iron was therefore recorded by a galvanometer connected in series with the thermocouple system.

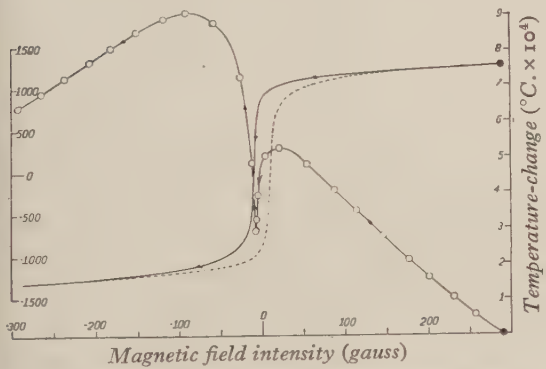


Fig. *A*. The magneto-thermal relations associated with the largest hysteresis loop (Ellwood).

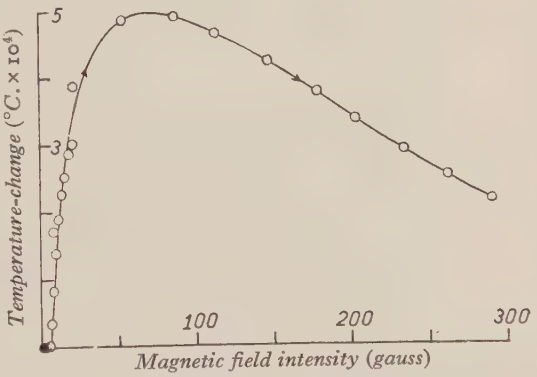


Fig. *B*. The magneto-thermal phenomena of the virgin curve.

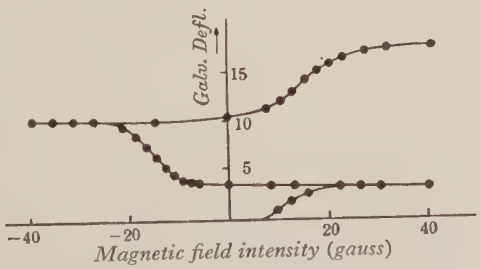


Fig. *C*. Honda's curve for carbon steel (0·2 per cent. C).

Figure *A* shows the curves which Ellwood obtained in the case of steel containing 1·08 per cent. of carbon. In this figure we have the hysteresis curve for this steel and the curve showing the observed temperature-changes as the magnetizing field changed from + 290 to − 290 gauss. As the field is reduced from 290 to 20 gauss a steady rise in temperature is observed. On reduction of the field below + 20 gauss a cooling effect is observed, and this cooling persists until a field of − 8 gauss is attained. Between − 8 and − 90 gauss a sudden rise in temperature is observed, and finally between − 90 and − 290 a gradual cooling effect. The coercive force was approximately 8·4 gauss.

The temperature-changes recorded for the same specimen when it is first thoroughly demagnetized and later magnetized in a field which is gradually increased to 290 gauss are shown in figure *B*. Now, such curves have been obtained so far for two steels containing 1·08 and 1·35 per cent. of carbon respectively, and we do not know whether they are to be observed with other specimens. It is true that Honda* has found no evidence of these cooling-effects with the carbon, tungsten and Japanese magnet steels which he has used. His results for a carbon steel containing 0·2 per cent. of carbon are reproduced in figure *C*. It should be mentioned, however, that Honda's apparatus was very much less sensitive than Ellwood's, so that, as far as one can tell from the available data, we are only justified in stating that such cooling phenomena are observed with two steels containing the stated percentages of carbon.

Now it seems very reasonable to suppose that the phenomena shown in figure 5 of the paper under discussion are intimately connected with thermal changes such as are found by Ellwood. That is why, in my opinion, it is desirable to have complete information concerning the carbon content and the hysteresis properties of the iron used in these thermoelectric experiments. As far as I am aware, no explanation of the cooling phenomena described by Ellwood has yet been given. I hope that the authors will continue their experiments, and I hope they will have the good fortune to obtain an explanation of these complicated thermomagnetic effects.

Note received from Authors. Commercial nickel and iron wires were used, their purity being guaranteed as 99·5 per cent.

* K. Honda, J. Okubo and T. Hirone, *Sci. Rep. Tokio University*, **18**, 409 (1929).

# ON PERIODIC MOVEMENTS OF THE NEGATIVE GLOW IN DISCHARGE TUBES

By W. A. LEYSHON, Ph.D.

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*ABSTRACT.* An account is given of experiments designed to discover the cause of the jumping-glow phenomenon in discharge tubes. It is shown that the effect is produced when traces of hydrocarbon vapour are present in the tube in addition to the filling gas (neon, in most of the experiments here described). Some effects of such traces of vapour on the characteristics of the tube are given.

It is suggested that the phenomenon of the jumping glow may be due to internal flashing at that part of the surface of the cathode which is not covered by negative glow. The flash may be caused by the electrical breakdown of a partially insulating hydrocarbon layer, as a result of the collection of positive ions, or by a surface chemical action occurring when the reaction-products of the discharge have reached a certain concentration at that surface. It is supposed that the electron emissivity of the surface is increased by the flash, and that the main glow jumps to the activated surface. The process is reversible and hence may be periodic. Observations connecting the periodic time of jumps with the current through the tube are in support of this explanation, as also are other experimental results described in the paper.

## § 1. INTRODUCTION

Periodic movements of the negative glow in a neon discharge tube were first observed in December 1929.

As a result of some previous observations on the characteristics of discharge tubes under flashing conditions[1], it was decided to experiment with neon tubes of the Osglim I type but having very small anodes. Two of these tubes, filled with 98 per cent. neon and 2 per cent. helium at pressure of 14·7 mm. of mercury and free from the impurities usually present in tubes of the commercial type, were supplied by the Research Department of the General Electric Company. (The electrodes, as in the ordinary Osglim lamps, were of iron.) On attempting to obtain their volt ampère characteristics, it was found that for certain adjustments of current and voltage the negative glow began to execute periodic movements over the surface of the cathodes. This periodic movement was demonstrated to the Physical Society and a short account was given[2] of the observations which had been made on the two lamps. Later, four other lamps of somewhat similar construction were obtained, two of them filled with neon at 7·7 mm. pressure, and these also showed periodic movements of the negative glow under suitable conditions. Some of these were demonstrated at the Royal Institution soirée on January 23, 1931.

A number of commercial tubes were tested, but in one tube only was the effect observed, and in that only when the wire and not when the plate was used as cathode.

Figure 1 shows the circuit used: in the first experiments made the diode was replaced by a resistance of the order of 100,000 ohms. Figure 2 illustrates the regularity with which these movements were carried out. Figure 3 illustrates some of the effects observed.

In these early observations two distinct types of movement were recorded. In the first, which occurred with larger values of current and voltage, the glow was reddish and the movements were slow, apparently unaffected by transverse magnetic

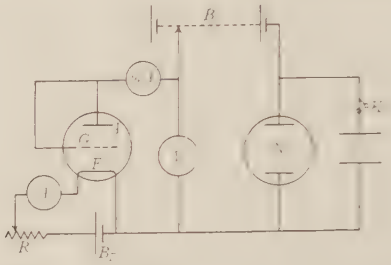


Fig. 1. *N*, neon lamp; *B*, battery; *mA*, milliammeter; *A*, *G*, *F*, triode used as diode; *A*, anode; *F*, filament; *G*, grid; $B_F$, filament battery; *A*, ammeter; *R*, filament resistance; *K*, key; *V*, electrostatic voltmeter; *C*, condenser.

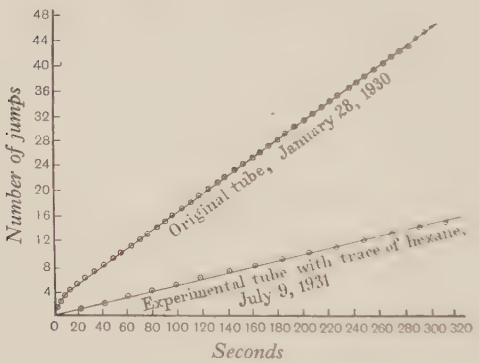



Fig. 2.

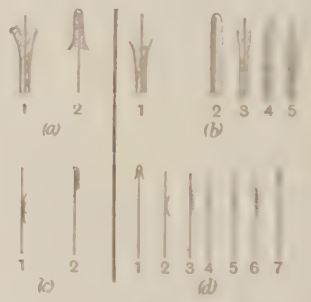



Fig. 3.

Fig. 3. (*a*) and (*c*) pressure 7·7 mm.; (*b*) pressure 14 mm.; (*d*) commercial Osglim I type, small electrode as cathode; (*c*) 1 and (*d*) 3 and 6 indicate very rapid flickers. In (*b*) the distance between the diagrams illustrates relative time-intervals. The cathode is viewed edgewise in all diagrams.

fields; increase of applied potential increased the frequency of movement; heating the tube* by radiation increased the frequency; the changes in current through the tube accompanying the jumps were small. This type only was seen in the tubes containing neon at 7·7 mm. pressure.

The second type of movement occurred almost at the extinction point of the lamp. The glow was much whiter and smaller; it moved very rapidly; the movement was appreciably affected in frequency by a transverse magnetic field; increasing the applied potential slightly slowed up the movement; heating the tube* slowed up the movement; the current changes during the cycle were comparatively large.

* Heating the tube probably liberated adsorbed material from the glass walls.

After some days of use the lamp which had shown the second type of oscillation did so no longer. Both types of movement altered to some extent with time and in such a way that the periodic time tended to become longer as the tube was used; also the adjustments necessary to obtain the jumps became more critical. If the tubes were left unused they did not appear to alter much with time.

Another set of tubes supplied by the General Electric Company with a different electrode design showed no jumping movement at all. One of these tubes contained helium.

## § 2. SOME FURTHER OBSERVATIONS ON THE ORIGINAL TUBES

Some observations were made on the current changes taking place through one tube during the movements of the glow. The circuit is shown in figure 4. A Gambrell short-period galvanometer with a sensitivity of 0·25 $\mu$A per division was used. It was balanced so that only the variations in the current through the neon tube were indicated.

It was observed under some circuit conditions that just before a jump took place small periodic variations of the current occurred, and increased in amplitude until finally the glow jumped; at this stage the largest change in current was observed. These observations were repeated later, a cathode-ray oscillograph being used to indicate changes in voltage across the tube. The circuit is shown in figure 5.

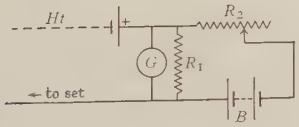



Fig. 4.

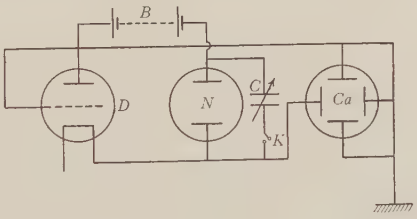



Fig. 5.

Fig. 4. *Ht*, high-tension battery of set; $R_1$, $R_2$, resistances; *G*, galvanometer; *B*, battery.

Fig. 5. *N*, lamp; *B*, battery; *Ca*, cathode-ray oscillograph; *D*, diode; *C*, condenser; *K*, key.

Again, small periodic movements of the spot on the fluorescent screen were sometimes observed before the jump took place; when this jump actually occurred, a larger displacement of the spot was seen.

On other occasions the oscillograph spot gave one rapid movement as if a small flash had occurred in the lamp. Sometimes after a jump had occurred the spot would move slowly towards the zero position corresponding to smaller current through the lamp, and then jerk back suddenly as the return jump occurred.

Similar observations were made with the oscillograph on a neon-filled experimental tube, with aluminium electrodes, described in § 4.

*Phenomena observed with condensers in parallel with the discharge tube.* When, in the circuit of figure 1, a condenser was put in parallel with a tube which showed the jumping-glow effect, with suitable values of current and voltage the tube flashed once or twice, then a steady glow was seen which moved to a new position on the cathode and disappeared; the whole process was then repeated. With a suitable value of condenser capacity and very careful adjustment of diode-filament current it was possible to obtain a condition in which the glow moved very rapidly, with the second type of movement seen previously in one of the original tubes with no condenser in parallel. The rapidity of movement of the glow was affected as in the previous case by the presence of a transverse magnetic field. With larger values of the condenser and a greater current through the diode it was possible to get the following sequence: flash (whitish glow all over cathode), darkness, steady reddish glow which slipped, darkness; and then the cycle repeated. The current was in each case near the critical current for flashing. With a still greater capacity and an apparently steady glow, a further increase in capacity altered the appearance of the glow and a quick ripple appeared at the top. With suitable adjustments this flicker became so great in extent that the top reached the end of the cathode and then the glow contracted and jumped to the end of the cathode. After a time it dropped again, the flicker appeared and the whole cycle was repeated. This effect was seen later in other neon-filled tubes without external capacity, and also in a hydrogen-filled tube. The value of the parallel capacity necessary to produce these effects depended on the tube used and the current at which jumps tended to occur. In one tube in which these results were obtained with capacities of the order of 0·002 $\mu$F, the corresponding movements of a cathode-ray-oscillograph spot were noted. The plates of the oscillograph were connected in parallel with the diode. For a particular value of current through the tube flashing of the ordinary type occurred for medium values of the condenser. A faint line then covered the whole diameter of the fluorescent screen of the oscillograph. On reduction of the condenser capacity the faint line shortened and became brighter, and usually shortened gradually until only the bright spot was seen. With careful adjustment, however, the flickering effect could be produced, and then the faint line with a superposed bright line was seen on the screen of the oscillograph. If a jump occurred, and the glow contracted to the top of the cathode, the line on the screen disappeared and the spot appeared steady. No doubt this effect is connected with the high note often heard on reduction of the current, just before flashing occurred. It may also have some relation to the saw-tooth oscillations described by Finch, Sutton and Tooke[3]. The tube containing helium showed the flicker effect with a condenser in parallel, but no jump.

One of the original tubes was opened to the air, fitted with a tube and tap and connected to a Hyvac pump. When the pressure had been sufficiently reduced the tap was closed, and the tube was included in the circuit of figure 1. An exceedingly slow periodic movement of the negative glow was observed on one occasion only: considerable hysteresis in the extent of the glow was, however, noticed on first increasing and then decreasing the current through the tube.

Taylor[4] observed a periodic movement of the negative glow in a beehive Osglim lamp filled with air; he ascribed the phenomenon to a gas-film at the cathode surface[5].

## § 3. EXPERIMENTS WITH HYDROGEN

In order to be able to examine the jumping effect in more detail, it seemed desirable to have an experimental tube into which different gases could be introduced. The tube mentioned above was connected up to a vacuum apparatus arranged as suggested in figure 6. After being thoroughly evacuated (but not baked out nor having its electrodes degassed) the tube was filled with hydrogen obtained either from a palladium tube or by electrolysis of dilute sulphuric acid. In the latter case the hydrogen was passed over phosphorus pentoxide and through a liquid-air trap before being admitted to the experimental tube. On being tested in the circuit of figure 3 the tube showed very rapid flickers and jumps, much more rapid than any seen before. The outline of the negative glow was in many cases, and particularly at higher pressures, very suggestive of that of a liquid on a greasy surface. If the current through the tube was increased the appearance of the end of the glow often suggested drop-formation, and on some occasions a drop of glow gradually removed itself from the main glow. Figure 7 illustrates this effect, which was also seen in some later experiments with neon. The periodic movements of the negative glow in hydrogen were not as regular nor as persistent as those previously seen, and were usually accompanied by audio-frequency oscillations.

## § 4. EXPERIMENTAL TUBE WITH ADJUSTABLE ELECTRODE

In order to see whether the jumping effect was due to the small size of the anode an experimental tube was used, figure 8, in which the effective length of the wire anode was adjustable. The electrodes were of pure nickel, and at first the wire anode was of the same length as the plate cathode. The tube was joined to the vacuum apparatus, figure 6, which had been modified by the addition of a glass container filled with 98 per cent. of neon and 2 per cent. of helium.

After being evacuated thoroughly, the tube was filled with neon at a pressure of 10 mm. of mercury, and then connected up with the circuit of figure 1. Practically all the phenomena previously seen in the neon tubes were found to occur with this tube. On re-evacuation and refilling with neon, however, the jumping-effect could not be reproduced.

In order to see whether the jumps were more readily obtainable with a small anode, the sealing-wax joint at *S*, figure 8, was gently heated with a small flame, and the anode was pulled down so that only a small piece projected above the glass. Rather irregular jumps were obtained.

In a further series of experiments it was found possible to rid the tube of the cause of the jumping glow, by passing a discharge through it from a small induction

coil and simultaneously heating the glass tube while evacuating. On filling the tube with neon and testing after such treatment it was usually found impossible to persuade the glow to jump, even by the most careful adjustment of current and voltage.

After the wax joint had been softened and the anode wire pushed up, so that again the anode was of the same length as the cathode, it was found that very vigorous jumps could be obtained, either electrode being used as cathode. It was thus clear that the effects were not due to the smallness of the anode in the original tubes. This was confirmed by the fact that one lamp of the commercial I type showed a jumping effect, the wire being used as cathode.

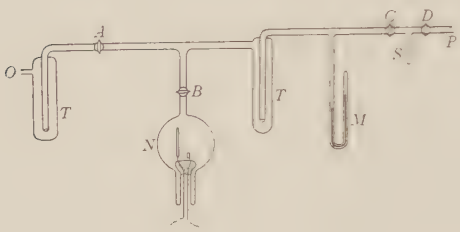



Fig. 6.

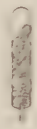


Fig. 7.

Fig. 8.

Fig. 6. *N*, experimental tube; *M*, mercury manometer; *S*, side tube (or palladium tube); *T*, *T*, liquid-air traps; *A*, *B*, *C*, *D*, taps; *P*, tube leading to gas reservoir; *O*, tube leading to McLeod gauge and pump system.

Fig. 7. (Shaded part indicates negative glow.)

Fig. 8. *T*, glass tube; *P*, plate; *B*, brass cap; *L*, *L*, leads; *W*, wire; *G*, *G*, glass guide-tubes; *S*, *S*, sealing-wax joints.

## § 5. INTRODUCTION OF TRACES OF OTHER GASES OR VAPOURS INTO THE DISCHARGE TUBE

It was noticeable in the experiments just described that jumps could always be obtained after the adjustment in length of the anode wire. This involved heating the sealing-wax joint and it was possible that a little air might have entered the experimental tube.

In order to find out whether traces of oxygen were responsible for the effect, this gas was deliberately introduced by heating potassium chlorate in a side tube. In the first experiment of this kind the potassium chlorate had been slightly contaminated with tap grease. It was found that the jumping effect which had been eliminated by the method previously described was in all cases restored if the potassium chlorate was heated, not too violently. After the heating of the potassium chlorate a higher value of applied voltage was necessary to produce the glow; also flashing occurred more readily.

The next step was to use pure potassium chlorate, uncontaminated by grease. The experimental tube, electrodes and side tube were removed and thoroughly

cleaned, pure potassium chlorate was introduced into the side tube and the degassed electrodes were fitted again into the experimental tube. (Apiezon tap grease was now used for the taps.) It was found that the jumping effect, after elimination in the usual way, could not be restored by heating the potassium chlorate.

Air was then let in through tap *C*, figure 6, the pressure was reduced to about $10^{-3}$ mm. and neon was introduced. No jumps were obtained on testing. After refilling with pure neon, it was found that on reduction of the effective length of the anode (by heating the sealing-wax joint, and adjusting) the jumps were restored. In pushing the anode up again, some sealing-wax was introduced into the small glass guide tube for the anode above the joint. It was found in a series of experiments that jumps could always be restored by gently heating this sealing-wax. Either electrode used as cathode showed the effect. Thus it appeared that the jumping effect could be restored to the tube by introducing volatile products obtained by heating tap grease or sealing-wax. A comparison of the chemical composition of tap grease and sealing-wax suggested that the effect was possibly produced by a hydrocarbon of the terpene series.

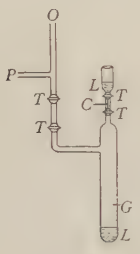



Fig. 9.

Fig. 10.

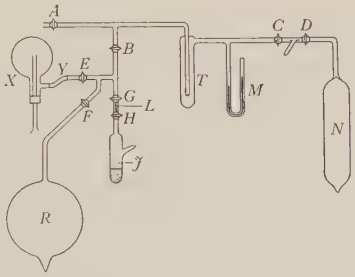



Fig. 11.

Fig. 9. *G*, glass tube; *C*, capillary tube; *O*, tube leading to vacuum apparatus; *T*, *T*, taps; *L*, limonene; *P*, tube leading to experimental tube.

Fig. 10. *P*, plate cathode viewed edgewise; *W*, wire anode; *G*, glow (jumping to positions shown dotted).

Fig. 11. *A–H*, taps; *T*, liquid-air trap; *X*, experimental tube; *N*, neon reservoir; *J*, hexane-container; *Y*, connecting-tube; *M*, mercury manometer; *R*, reservoir; *L*, capillary tube.

Accordingly the apparatus was modified so that a volatile terpene, limonene, could be introduced into the experimental tube by means of a side tube, figure 9.

No very marked effect was produced on the introduction of a small quantity of vapour. The glow was smaller and whiter, but no jumps were obtained. If, however, the apparatus was evacuated, limonene vapour introduced, and an induction coil discharge passed while re-evacuating, vigorous jumps were obtained on filling with neon and testing as before. The limonene vapour in the tube did not alone produce the jumping effect; the latter appeared to be due to a modification of the surface of the cathode or of the gas content of the tube as a result of sending a discharge through limonene vapour.

A very significant type of jumping glow was produced in the tube thus prepared: see figure 10. The small glow in the centre of the cathode jumped up and

down at a rate which could be varied from about 2 oscillations per second to one corresponding to a note above audible frequency in the telephones in the circuit. At the slow rate, it was easily seen that a small flash occurred above or below the glow almost simultaneously with its movement up or down.

On removal of the nickel electrodes, the plate cathode was seen to be covered with a brownish deposit on the side nearest the anode. The other side appeared greenish. The electrode when looked at under a strong lens was seen to be pitted all over its surface. The nickel electrodes were now replaced by clean degassed aluminium electrodes of the same shape and size as the original nickel electrodes. They gave similar effects to those noted in the case of nickel, but the jumping effect was much more difficult to remove.

On one occasion, with the minimum possible applied voltage and a small current through the tube, a faint glow was observable in the neighbourhood of the wire anode. A bright flash, which seemed to move downwards over the surface of the plate cathode, appeared at regular intervals; with small current about every 7 seconds. (There was no external capacity in parallel with the tube.) On increasing the current this flash could be made to occur at audible frequency. With sufficient limonene in the tube, a jumping effect similar to that illustrated in figure 10 was obtained.

The experimental tube was removed from the apparatus and thoroughly cleaned and baked out; it was fitted with clean degassed nickel electrodes and replaced. After thorough evacuation, neon was introduced at 12 mm. pressure and the tube was tested. There was a slight tendency for the glow to jump, but this was removed by the usual process of heating the glass and simultaneously passing a discharge from an induction coil while evacuating. After refilling, toluene was introduced as the limonene had been previously. Tap *B*, figure 6, was left open to the liquid-air trap, to remove excess toluene. A high voltage was required to start the discharge and a purplish glow was seen which moved slowly up and down the cathode. This occurred whether the plate or the wire was used as cathode.

The movement was far less jerky than any seen before. When the tube was thoroughly evacuated, and then was refilled with neon at a pressure of 5–6 mm., the glow showed no tendency to jump. An induction coil discharge had not been passed through the tube, nor had the glass walls been heated. Toluene was again introduced, the total pressure was reduced to about 0·04 mm., and more neon was let in. Very vigorous jumps were observed at almost all values of the current for which the discharge did not cover the cathode. The experiments were repeated, greater precautions having been taken to eliminate air. Very vigorous jumps and flashing effects were seen, and an oily deposit was visible on the cathode.

A new tube fitted with degassed nickel electrodes was now used. Hexane was introduced in the same way as limonene and toluene had been previously. The jumping glow was obtained at once. Introduction of more hexane vapour made the glow contract very much and the cathode became blackened.

The method so far used of introducing hydrocarbon vapour into the discharge tube had not been entirely satisfactory, since a little air and some tap grease might

enter with the liquid into the reservoir. Further, it seemed desirable to have a more definite idea of the amount of hydrocarbon vapour introduced into the experimental tube. The apparatus was accordingly modified as shown in figure 11, and the whole was thoroughly evacuated with liquid air on the hexane container, the electrodes having been removed and cleaned and replaced so that wire and plate were of the same length. Considerable difficulty was found in getting rid of jumps, owing to previous absorption of the hexane by the glass of the experimental tube. However, this was accomplished finally by passing an induction-coil discharge while evacuating the tube without heating the glass.

Characteristics of the tube, the plate and wire being used in turn as cathode, were then obtained with the apparatus of figure 1. This was done for five different pressures of neon. The pressure of the neon was adjusted to 14·5 mm. in the experimental and connecting tubes and reservoir, and the characteristic was determined before and after the introduction of a dose of hexane. The hexane-container was immersed in a freezing mixture consisting of solid carbon dioxide and ether, the tap $H$ was opened, $G$ being closed; then $H$ was closed and $G$ opened, $F$ being opened and $B$ and $E$ shut; and finally $E$ was opened. Characteristics, which were in all cases determined by starting with the highest value of current recorded and then reducing it, were also obtained after further doses of hexane had been thus introduced.

From the dimensions of the apparatus, and a knowledge of the vapour pressure of hexane at $-78°$ C., it was calculated that the partial pressure of hexane in the experimental tube after the first dose could not have exceeded $2 \times 10^{-5}$ mm. and was probably considerably less than this, since the taps were of small aperture and were not left open for more than a few seconds. Moreover, some of the introduced hexane vapour would be absorbed by the glass.

## § 6. EFFECT OF THE INTRODUCTION OF HEXANE ON THE CHARACTERISTIC CURVE AND GENERAL BEHAVIOUR OF THE DISCHARGE TUBE

Figures 12 and 13 show the current/voltage characteristics, of the tube, for wire and plate cathodes respectively, before the introduction of the first dose of hexane. Inset are curves connecting the minimum voltage reached with the pressure of gas.

Figures 14 and 15 show the effect of successive doses of hexane on the characteristic, for wire and plate cathodes respectively.

Figure 16 shows the effect of pressure on the value of the current for which the glow just fails to cover the cathode. Inset is the effect of hexane on this critical current in the case of the wire cathode. Figure 17 shows graphically the currents for which flashing started and stopped, for the series of pressures for which characteristics of the tube were obtained. There was no external capacity in parallel with the electrodes. The effect of introduced hexane on the hysteresis is indicated also. It is clear that hysteresis is increased (markedly in the case of the plate cathode) by the introduction of hexane.

From the results shown in the figures, it appears that a small quantity of introduced hexane has the effect of (*a*) lowering the voltage required to maintain a given current through the tube and altering the shape of the characteristic; (*b*) in-

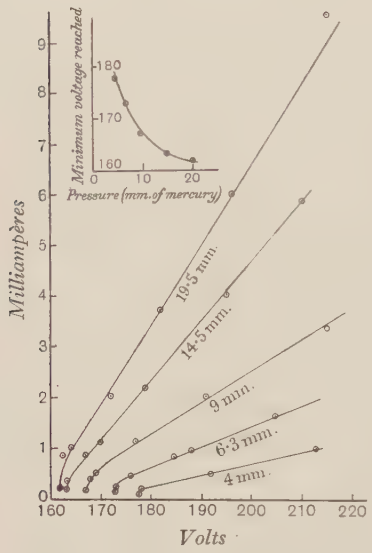



Fig. 12. Effect of pressure on characteristic. (Wire cathode.)

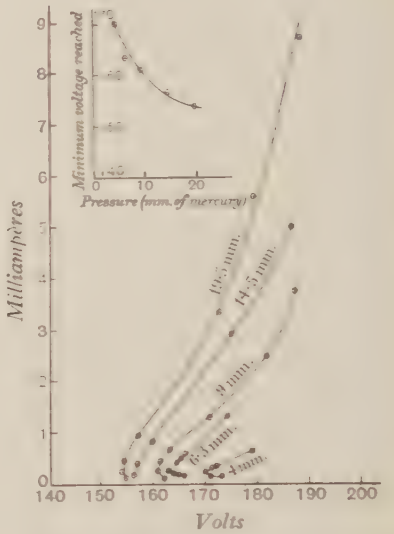



Fig. 13. Effect of pressure on characteristic. (Plate cathode.)

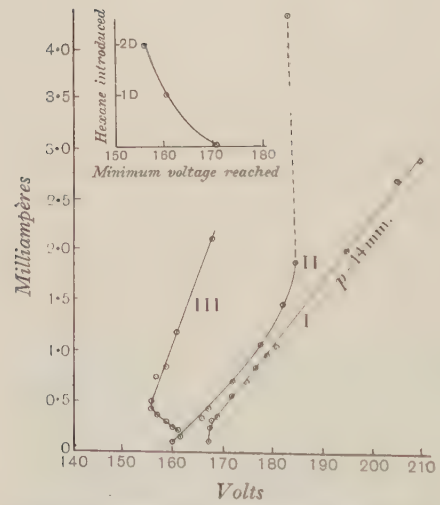



Fig. 14. Effect of introduced hexane on characteristic. (Wire cathode.) I, before introduction of hexane; II, 1st dose; III, 2nd dose.

Fig. 15. Effect of introduced hexane on plate characteristic. I, no hexane; II, two doses; III, further dose.

creasing the value of the current at which the glow just fails to cover the cathode; (*c*) causing jumping of the negative glow when the discharge does not cover the cathode; (*d*) increasing the hysteresis for flashing of the tube, there being no external capacity in parallel with it.

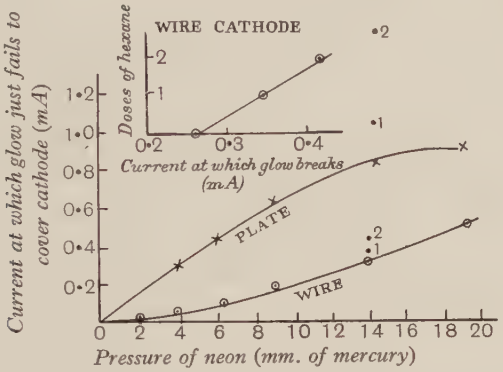



Fig. 16. 1, one dose of hexane; 2, two doses of hexane.

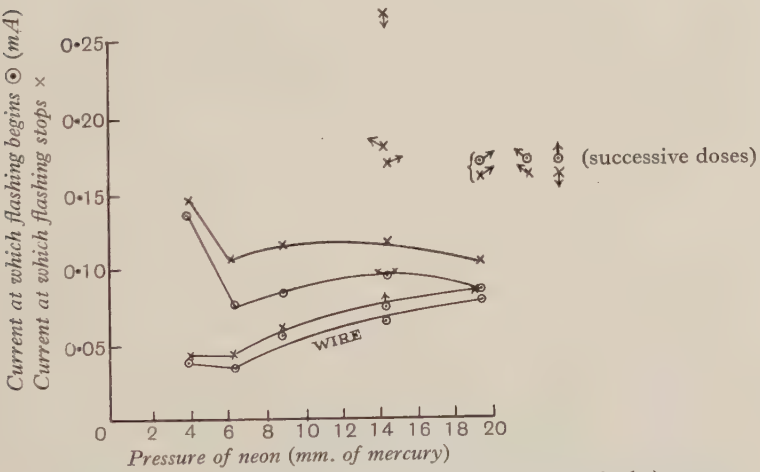



Fig. 17. Effect of successive doses of hexane on hysteresis. (Plate cathode.)

## § 7. DISCUSSION OF RESULTS AND OF POSSIBLE CAUSES OF THE JUMPING GLOW PHENOMENON

The results of the experiments described in the preceding sections suggest that the jumping-glow phenomenon is not due to a particular shape, disposition or chemical composition of the electrodes, nor is it, apparently, confined to a discharge through a gas of a particular molecular structure. It occurs, however, when minute quantities of certain hydrocarbon vapours are present in the discharge tube.

Some current/voltage curves for tubes which showed the jumping-effect are given in figures 18 and 19, in order to illustrate the characteristic discontinuities

of such curves. When a jump took place the voltage across the tube sometimes changed appreciably, on occasions by as much as 10 volts, but reverted to its original value on the return jump.

In an attempt to gain more insight into the mechanism of the phenomenon, a number of observations were made on the relation between the periodic time of jump and the current passing through the discharge tube. Some of these are shown in figures 20 (*a*) and (*b*). If a periodic-time current curve was taken before and after a reversal of current, its general character was on some occasions found to be altered considerably.

The curve marked *X*, figure 20 (*a*), was taken while flashing of the ordinary type occurred, no external capacity being in parallel with the electrodes. Jumps obtained during flashing sometimes took place in two or more stages, so that a little repeated tune was heard in the telephones attached to the circuit. Increase of current raised the pitch of all the notes, and so usually left the musical phrase unaltered but raised it to a higher key.

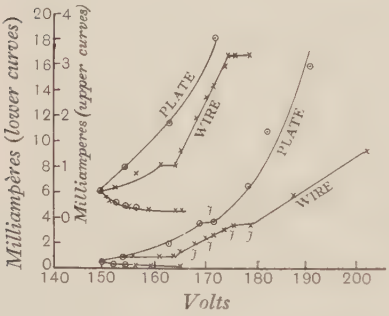



Fig. 18. Pressure 34·5 mm. of mercury. *J*, jumps.

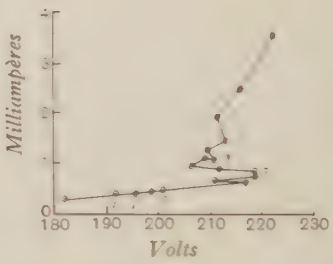



Fig. 19. Plate cathode, small anode. Pressure 10 mm. mercury. *J*,*J*, jumps; *F*,*F*, flickers.

The jumping phenomenon may be caused by the influence of the hydrocarbon vapour on any of the factors which contribute to the maintenance of a steady glow-discharge. The shifting of the characteristic curve in neon, figures 1 and 15, in the direction of smaller voltage may be partly accounted for by the additional energy available as a result of collision of hydrocarbon molecules with metastable neon atoms; the formation of slow-moving negative ions may account for the very marked diminution in the size of the glow on the introduction of traces of hydrocarbon vapour. The electrons which must be supplied in a steady discharge to make up for losses due to recombination and collection of charges by the glass walls are, it is usually supposed, obtained from the cathode, although ionization by positive ions has also been suggested by Townsend as a possible source of them.

The whole discharge takes place in such a way that the necessary electrons are produced with the minimum expenditure of energy[6]. They may be ejected from the cathode thermionically, as a result of local heating due to the impact of positive ions on the cathode surface; by photoelectric emission, due to the light given out

by positive ions in recombining with electrons, or in processes of ionization occurring in the discharge; or by autoelectronic emission due to the intense local electric field of an approaching positive ion[7]. If there is any possibility of chemical action at the cathode surface, the energy liberated by this action may also free electrons from the cathode[8].

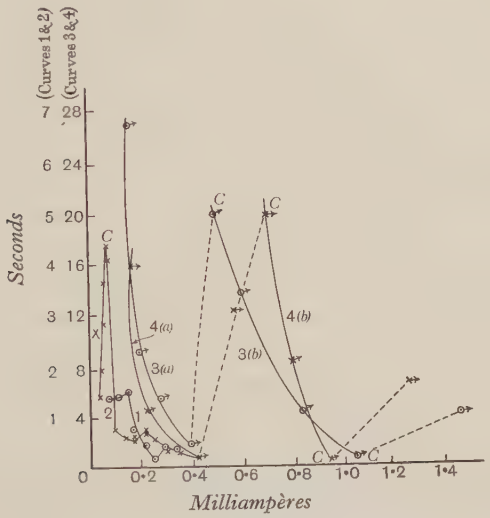
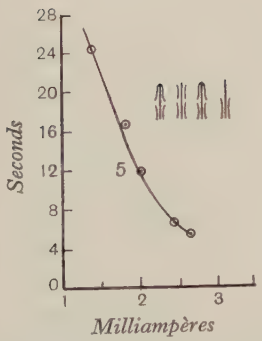
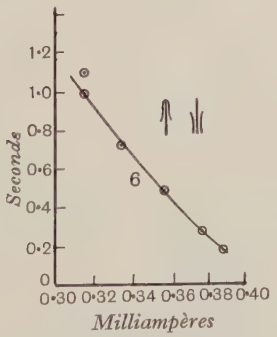


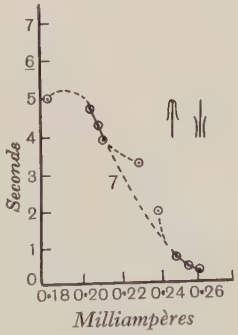

Fig. 20 (*a*). Curves 1 and 2, wire cathode. Curves 3 and 4, plate cathode. *C* indicates change in type of jump.




$p$ = 34·5 mm. Plate anode. Wire cathode. Diode in series.




$p$ = 7·5 mm. Small anode. Plate cathode. Resistance in series.




$p$ = 4 mm. Small anode. Plate cathode. Diode in series.

Fig. 20 (*b*).

The ease with which electrons are liberated from the cathode depends very largely on its surface condition; this is well known in the cases of photoelectric and thermionic emission, and it has recently been demonstrated in the case of autoelectronic emission[7]. Langmuir[10] has shown that a surface in a gas (or

vapour) tends to become covered with a monomolecular layer of the gas which is then said to be adsorbed on the surface*.

In any gaseous discharge it is extremely likely that there will be a film of gas formed on the cathode surface, since positive ions are continually arriving and being neutralized there. The extent to which the film will approach to a complete monomolecular layer will depend on the rate of evaporation, which may be altered profoundly by the discharge itself, and on the chemical interaction between the metal and the adsorbed atom or ion. If the film consists in a layer of adsorbed positive ions, this layer may behave as a positive grid[12] and assist materially in the ejection of electrons from the cathode, so lowering the sparking potential in the gas.

S. C. Lind[13], in a study of the action of $\alpha$ rays and of electrical discharges in gaseous hydrocarbons of the paraffin series, has shown that, in both cases, hydrocarbons of higher molecular weight are obtained, hydrogen and methane being liberated. He ascribed the process to an initial formation of ion clusters, with subsequent condensation to the compounds of higher molecular weight. E. G. Linder[14] found that for the glow discharge in paraffin hydrocarbons a similar chemical action occurred; he found further that if the products of the reaction were rapidly removed, the amount of chemical action in a given time was proportional to the current through the tube. In discussing his results, with those of other investigators, he concluded that most of the chemical action took place in the gas phase, but both he and Lind found that the cathode gradually became covered with a wax-like material insoluble in all organic reagents. This finally became carbonized if the discharge was continued. Similar effects have been observed in the course of the present investigation when the pressure of hydrocarbon vapour in the discharge tube was comparatively high.

We may suppose that in neon or other discharge tubes containing a small percentage of hydrocarbon vapour the chemical actions described above also take place, but to a greatly diminished extent owing to the smallness of the partial pressure of the vapour. If we assume that the chemical action occurs in the neighbourhood of the negative glow, then hydrogen, methane and heavy hydrocarbon ions may accompany the other positive ions on their journey towards the cathode; of these the more slowly moving heavy hydrocarbon ions are most likely to be neutralized before arriving at the cathode surface[7].

Hydrocarbon vapour will condense on and be absorbed by the glass walls of the discharge tube, which also collect ions and electrons diffusing from the main discharge.

In attempting to form a theory of the jumping phenomenon the following possible processes suggest themselves: (*a*) The average work function of the composite surface directly supplying the negative glow might be increased by the action of the discharge, owing to deposition of hydrocarbon molecules; when the glow jumped to a part of the cathode where the effective work function was less, the original surface would lose these molecules by evaporation. The periodic time

* An account of recent work on the properties of surface layers has been given by Adam[11].

would then be determined by the rate of evaporation and the rate of deposition of hydrocarbon molecules. The work functions of the two patches of surface would change at rates that would be independent of the current strength if we may assume that the area of the cathode covered by the glow is proportional to the current strength, and if temperature variations can be ignored. Hence the periodic time of jump would be independent of current strength, which is certainly not the case in the experiments here described (see figure 20).

(*b*) The effect might be due to collection of ions by the glass walls. The regularity of the jumping glow, and the very small effect (if any) on it of bringing earth-connected conductors in contact with the walls, discounts this hypothesis.

(*c*) The effect might be due to the slow chemical changes going on in the discharge, a sudden change occurring when the reaction had reached a given stage. If we assume that the amount of chemical action is proportional to the quantity of electricity transferred between the electrodes, then the periodic time of jump would be inversely proportional to the current. The results obtained are not in accord with this relationship.

(*d*) The part of the cathode not covered with glow might become covered with a more or less insulating layer of hydrocarbon molecules. Positive ions diffusing from the main glow would form a layer on this, and as they collected the potential difference across the layer would increase, until finally the breakdown voltage of the layer was reached. A flash would then occur, and the cleaned cathode surface might then have a smaller work function than the original part covered by glow, which would accordingly jump to the new position. The process would be reversible so that a periodic jump might result.

Suppose that a fraction $x$, proportional to the current strength $i$, of the cathode is covered with glow; $(1 - x)$ will be covered by the insulating layer. Further, suppose that the rate of collection of positive ions by the insulating layer is proportional to the current strength, and that the breakdown voltage $V$ of the insulating layer is a constant quantity.

Then $VC = \sigma$ where $\sigma$ is the surface density of positive charge on the insulating layer for which breakdown occurs, and $C$ is the capacity per unit area.

Also $\sigma = \alpha it/(1 - \beta i)$ where $\alpha$, $\beta$ are constants, $x = \beta i$, and $t$ is the time-interval between two successive jumps. Therefore curves connecting $it$ and $i$ should approximate to straight lines.

This is found to be the case for simple types of jump, when the two areas successively covered by glow at the beginning of the jump do not appreciably overlap. It is also found to be true for the more complicated types, provided that the actual type of jump does not change. If the latter happens, there is sometimes a break in the curve connecting $it$ with $i$ and another straight line results; see figure 21. This explanation of the jumping-glow phenomenon is parallel to that given by Copeland[15] to account for his results on secondary-electron-emission from metal surfaces contaminated by oil films.

A somewhat similar explanation may account for the rapid movements of small glows in some of the tubes originally tested, and the effect seen in the experi-

mental tube with aluminium electrodes when the cathode was covered with an oily layer (see § 5).

We may here assume that a flash occurs in the immediate neighbourhood of the main glow which then moves to the new position, whereupon a flash occurs again, so that the movement is continued until the end of the electrode is reached, when the glow disappears and the tube lights up with the glow in the original position. The process is then repeated. In this case as in others local heating due to the main glow may play a part in the process. The mechanism of movement is reminiscent of the kind of action which is supposed to occur when a nerve impulse travels down a nerve as a result of electrical or other stimulation.

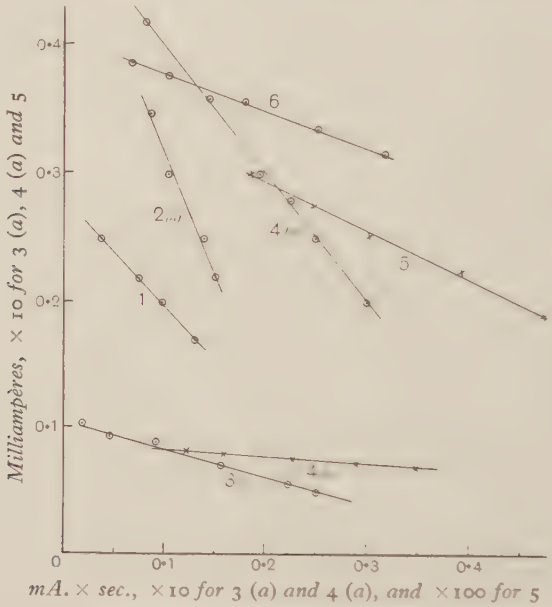



Fig. 21. Curves obtained from those of figures 20 (*a*) and (*b*).

The internal-flashing theory may also account for the flickers sometimes seen above the glow before a jump occurred, and it is upheld by the results of experiments with the short-period galvanometer and with the cathode-ray oscillograph, described in § 1. The theory also accounts for the very large range in frequency of jump observed under different conditions.

It is possible that the flash is due to surface chemical action occurring on that part of the cathode not covered by glow, when the reaction products of the discharge have reached a certain concentration in the neighbourhood of this surface. Such an explanation of the jumping glow would to some extent link up the phenomena with those of periodic chemical action[16].

When jumps occur during flashing, or while other audio-frequency oscillations are occurring in the tube, the same type of curve connecting *it* with *i* is not obtained; see figure 20 (*a*). In the former case, increasing the current increases the period of the jumping glow.

§ 8. ACKNOWLEDGMENTS

The experiments described have been carried out in the Physics Department of the London (R.F.H.) School of Medicine for Women. I should like to express my thanks to Dr W. H. Eccles, F.R.S., for lending me a Langmuir mercury condensation pump, and for the interest he has taken in the work; to Miss M. D. Waller for lending me a Hyvac oil pump, and a Gambrell short-period galvanometer; to the Council of the London (R.F.H.) School of Medicine for Women for grants from the Waller Memorial Research Fund for the purchase of apparatus, and to Mr W. Burrows of King's College for his help in making the experimental discharge tubes and setting up the vacuum apparatus.

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## DISCUSSION

Dr L. Simons remarked that Prof. O. W. Richardson, when baking out thermionic tubes by means of an alternating magnetic field, had found that some points on the walls of the tubes glowed brightly. When the glow had been removed by a strong heating it reappeared at other points. Possibly the effects described in the paper are due to a poisoning skin similar to that noticed by Prof. Richardson.

Mr P. B. Moon. I should like to ask Dr Leyshon whether the nature of the enclosing walls influenced the phenomenon. It seems possible that variation of the charge-distribution on the walls may affect the position of the discharge, and that hydrocarbon vapour may cause changes in the electrical properties of the glass surface.

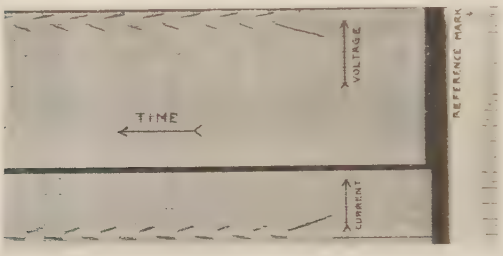


Fig. *A*. Current and voltage as functions of time.

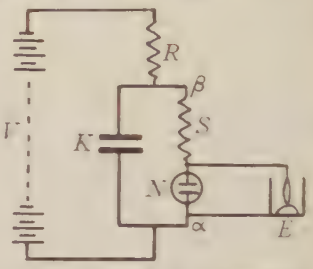


Fig. *B*. Circuit for measuring current and voltage.

Dr L. F. Richardson. The photograph, figure *A*, shows the course in time of the voltage and current when a neon lamp, instead of giving the usual alternation between flash and extinction, showed an alternation between two shapes of glow without any extinction.

The circuit, figure *B*, was that of Anson and Pearson with the additional resistance *S* and the Wulf electrometer *E*. The current was measured by transferring the electrometer contact from the point $\alpha$ to the point $\beta$. The time marks were 0·2 sec. apart. The voltage scale is $2{\cdot}01_6$ V per division. The current scale is $5{\cdot}4_2\mu$A per division.

The lamp was an Osglim with plate and spiral electrodes, the makers' coil having been removed. The glow was a small tuft fluttering to and fro on the spiral with about the period shown in figure *A*; so that we may suppose that the two slowly-changing parts of the record relate to the two positions of the tuft, and the sudden jumps of voltage and current relate to the sudden changes of position. It is seen that both in the jumps and in the slower changes an increase of current synchronizes with a decrease of voltage and vice versa. In one position of the glow the current rose slowly from 1 to 4 $\mu$A; in the other it fell slowly from 22 to 14 $\mu$A. The portion of the photograph on the right shows a reference mark at $156{\cdot}_3$ V from which we deduce that the extremes of voltage were $160{\cdot}_5$ and $151{\cdot}_5$ V in the periodic part of the record, which follows the introductory 0·6 sec.

The observed time-curves closely resemble a figure in B. van der Pol's paper on "Relaxation-oscillations"* showing a computed integral of his differential equation

$$\frac{d^2v}{dt^2} - \epsilon\,(1 - v^2)\,\frac{dv}{dt} + v = 0, \text{ when } \epsilon = 10,$$

* *Phil. Mag.* 2, 986 (1926).

$t$ being time, and $v$ in our case either voltage or current, both reckoned as deviations from their means.

*Further particulars.* $S = 0{\cdot}372$ megohm, mostly Ferranti wire-wound anode feed resistances, $R = 0{\cdot}93$ megohm (a Dumetome grid-leak), battery voltage $185{\cdot}_3$ V, from exide accumulators; $K = 3{\cdot}42\,\mu$F in good paper condensers. The electrometer had a period of 0·02 sec. and a damping ratio of $7{\cdot}_5 : 1$, so that detail briefer than 0·02 sec. should be ignored. The falling-plate camera was made by the Cambridge Instrument Co.

Author's reply. In reply to Dr Simons: it seems very likely that the effects he mentions as having been observed by Prof. O. W. Richardson are allied to those I have described.

Undoubtedly, as Mr Moon suggests, it is possible that variations in the electrical state of the walls of the tube due to the presence of adsorbed hydrocarbon may occur as the discharge proceeds, and that in this way the walls might produce a variable third-body effect on the main discharge between the metal electrodes. One would expect this effect to be most marked when the main discharge is a heavy one, and is comparatively near to the glass walls. I do not think, however, that the phenomena I have described are due to this cause, for the following reasons:

(1) In the original tubes, very little effect on the periodic time of jump is produced when earth-connected conductors are brought into contact with the outer walls of the tube; I tried this experiment some time ago at the suggestion of Mr J. E. Calthrop. Further, the glow cannot be induced to jump, unless it is nearly ready to do so, by bringing highly charged conductors near to the tube. (2) I have recently observed jumps, of the same type as those already described, in a large experimental tube, with greatly increased distance between the electrodes and the walls. (3) A visible flash at a part of the cathode surface often precedes the jump of the main glow to this part. (4) It is surprising to find, in spite of all the assumptions made in the simple theory put forward in the paper, that the prediction it gives of a linear relationship between $it$ and $i$ is found to be verified in most of the cases suitable for study. It is difficult to see how these results could be accounted for if varying changes on the glass walls are responsible for the phenomenon.

Dr L. F. Richardson's beautiful photograph, with its very close correspondence to a theoretical relaxation-oscillation curve given by Dr van der Pol, shows much more clearly than my original observations made in 1929 that the phenomenon is, as I suggested then, a form of relaxation oscillation.

Addendum by Dr L. F. Richardson. In view of Dr Leyshon's interesting researches on the effect of hydrocarbons, I have examined the spectrum of my Osglim lamp which showed the jumping glow. The spectrum shows no trace of the Swan bands of hydrocarbons. Neon, helium and argon are evident. The following faint lines suggest the presence of iodine: 5119, 5115, 4864, 4410, 3483 Å. The Balmer lines of hydrogen are absent, although they show plainly in four other Osglim lamps.

# A CATHODE-RAY OSCILLOGRAPHIC METHOD OF MEASURING INDUCTANCE

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Imperial College of Science and Technology

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*ABSTRACT.* The voltage fluctuations across the condenser in a damped oscillatory circuit, comprising inductance, capacity and resistance, are recorded by means of a cathode-ray oscillograph. The following relationship is derived:

$$L = CV_1^2\{1 + \lambda + (\tfrac{1}{2} + 2/\pi^2)\lambda^2 + (\tfrac{1}{6} + 2/\pi^2)\lambda^3\}/i_0^2,$$

where $L$ is the inductance, $C$ the capacity, $V_1$ the first peak voltage across $C$ in a train of damped oscillations, $i_0$ the primary current through $L$ at break of a charging circuit and $\lambda$ the logarithmic decrement of the oscillatory circuit. The measurement of $V$ and $\lambda$ from the oscillogram is discussed.

## § 1. INTRODUCTION

In connexion with a study of the coil ignition of gases, upon which we have recently been engaged, the need arose for some convenient means of measuring the values of the inductances of certain oscillatory circuits under actual working conditions. The frequencies of the circuits were unknown, nor would it have been convenient at the time to determine them.

In what follows an account is given of a cathode-ray oscillographic method by means of which the inductance of a coil may be measured, under working conditions, in terms of the value of the current flowing at the moment of break in a charging circuit and of the maximum voltage across a condenser of known value in an oscillatory circuit, without a knowledge of the natural frequency of the latter. Alternatively, if this frequency be known, the circuit inductance and capacity can both be determined.

## § 2. THEORY

*LCR* At any time after break, the general equation for the circuit $LCR$, figure 1, is

$$L\,di/dt + Ri + V = 0,$$

*V* where $V$ is the potential across $C$. Now

*i*
$$i = C\,dV/dt;$$

$$\therefore\ d^2V/dt^2 + (R/L)(dV/dt) + V/LC = 0.$$

When $R^2/4L^2 < 1/LC$, the solution is

*ωt*
$$V = e^{-kt}(A\cos\omega t + B\sin\omega t),$$

*k* where $k = R/2L$, and $\omega = (1/LC - R^2/4L^2)^{\frac{1}{2}}$.

On differentiating and substituting in $i = C\,dV/dt$, we have

$$i = Ce^{-kt}\{(B\omega - Ak)\cos\omega t - (A\omega + Bk)\sin\omega t\}.$$

The initial conditions are that, at break, $t = 0$, $i = i_0$ and $V = -Ri_0$, each being equal to $-E$, where $E$ is the e.m.f. of the battery. $i_0$ $E$

$$\therefore\ i_0 = C(B\omega - Ak), \qquad B, A$$

$$A = -Ri_0,$$

and

$$B = i_0(1 - R^2C/2L)/\omega C.$$

The expression for $i$ can be written

$$i = Ce^{-kt}\{(B\omega - Ak)^2 + (A\omega + Bk)^2\}^{\frac{1}{2}}\cos(\omega t + \theta)$$

$$= Ce^{-kt}\{(A^2 + B^2)(\omega^2 + k^2)\}^{\frac{1}{2}}\cos(\omega t + \theta),$$

where $\tan\theta = (A\omega + Bk)/(B\omega - Ak) = -R/2\omega L.$ $\theta$

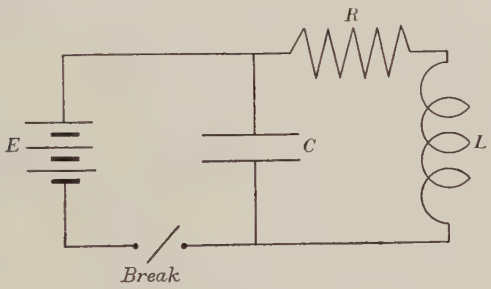


Fig. 1.

Now $V$ has its maximum values when $i = 0$, i.e. when $\cos(\omega t + \theta) = 0$, or $\omega t + \theta = \frac{1}{2}\pi, \frac{3}{2}\pi$, etc., or $\omega t = \frac{1}{2}\pi - \theta, \frac{3}{2}\pi - \theta$, etc. Therefore, the maximum values of $V$ are given by

$$V = e^{-kt}(A\sin\theta + B\cos\theta),$$

where

$$t = (\tfrac{1}{2}\pi - \theta)/\omega, (\tfrac{3}{2}\pi - \theta)/\omega, \text{ etc.},$$

$$= e^{-kt}(A^2 + B^2)\,\omega/\{(A^2 + B^2)(\omega^2 + k^2)\}^{\frac{1}{2}}.$$

Now

$$\omega^2 + k^2 = 1/LC,$$

and

$$A^2 + B^2 = i_0^2\{R^2C^2\omega^2 + (1 - R^2C/2L)^2\}/\omega^2C^2$$

$$= i_0^2/\omega^2C^2.$$

$$\therefore\ V = i_0(L/C)^{\frac{1}{2}}e^{-kt}.$$

If $V_1$, $V_2$ be the first and second peak values of $V$, then $V_1$, $V_2$

$$V_1 = i_0(L/C)^{\frac{1}{2}}e^{-k(\frac{1}{2}\pi - \theta)/\omega},$$

and

$$V_2 = i_0(L/C)^{\frac{1}{2}}e^{-k(\frac{3}{2}\pi - \theta)/\omega}.$$

$$\therefore\ V_1/V_2 = e^{k\pi/\omega},$$

and $\lambda$, the logarithmic decrement, is equal to $k\pi/\omega$.

$$\therefore\ V_1 = i_0(L/C)^{\frac{1}{2}}e^{-\lambda(\frac{1}{2}\pi - \theta)/\pi}.$$

Now, $\tan\theta = -R/2\omega L = -\lambda/\pi$; and, as $\theta$ is small, we can write

$$\tan\theta = \theta = -\lambda/\pi, \text{ neglecting } \lambda^3.$$

$$\therefore\ V_1 = i_0 (L/C)^{\frac{1}{2}} e^{-\frac{1}{2}\lambda - \lambda^2/\pi^2}, \text{ neglecting } \lambda^4$$

$$L = CV_1^2 e^{\lambda + 2\lambda^2/\pi^2} \div i_0^2$$

$$= CV_1^2 \{1 + \lambda + (\tfrac{1}{2} + 2/\pi^2)\lambda^2 + (\tfrac{1}{6} + 2/\pi^2)\lambda^3\}/i_0^2,$$

$\lambda^4$ being neglected. Thus the value of the inductance can be obtained from

Fig. 2.

measurements of the peak voltage across the condenser, $C$, of known capacity, and the logarithmic decrement of the oscillations as recorded on a cathode-ray oscillogram such as is shown in figure 2.

### § 3. EXPERIMENTAL METHOD

A cathode-ray oscillograph, operating in conjunction with the type of time base previously described*, does not in the case under consideration record a base line. Before the necessary measurements can be carried out, this line must be located. This may be done as follows:

In figure 2 let $a_1$, $a_2$, $a_3$, ... be the amplitudes of successive oscillations from the base line, the position of which is as yet unknown. The quantities which can be actually measured to a sufficiently high degree of approximation are $A_1$, $A_2$, etc., where $a_1, a_2, a_3$

$$A_1 = a_1 + a_2, \qquad A_1$$

$$A_2 = a_2 + a_3, \qquad A_2$$

.....

Now $$a_2/a_1 = a_3/a_2 = \ldots = l, \qquad l$$

$$\therefore A_1 = a_1 + a_1 l$$

$$= a_1 (1 + l),$$

and $$A_2 = a_1 l (1 + l).$$

$$\therefore A_2/A_1 = l;$$

from which the logarithmic decrement can be determined.

The value of $l$ having been thus determined, the values of $a_1$, $a_2$, $a_3$, etc. follow, because

$$a_1 = A_1/(1 + l).$$

Thus the base line may be located. In practice, for the purpose of measuring up an oscillogram, a suitable voltage deflection scale is employed, such as is shown in figure 3 of the previous paper*.

## DISCUSSION

Dr D. Owen. The theory worked out in the paper is based on the assumption of constancy of the inductance and resistance of the coil under test, and it also neglects the self-capacity of the coil. As regards the resistance term, the effective resistance to varying currents of an iron-cored coil may be very high compared with the ohmic resistance of the winding; and this extra resistance, arising from hysteresis and eddy currents, must be dependent on the current flowing as well as on the frequency. Unfortunately no actual example of results is given in the paper, and in the absence of such figures further commentary is barred.

Prof. Finch's success in the application of the cathode-ray oscillograph has been signally demonstrated in an earlier paper read before the Society, and doubtless the difficulties here referred to will be surmounted by the authors.

* Finch, Sutton and Tooke, *Proc. Phys. Soc.* **43**, 502 (1931).

Authors' reply. It is clear from the general circuit equation set forth at the beginning of § 2 that the theory is only applicable to cases where the self-capacity of the coil is negligible in comparison with $C$, and $R$ is independent of the current traversing the coil. Thus the presence of iron in the magnetic circuit can give rise to very considerable inaccuracy, for the reasons pointed out by Dr Owen. It is our intention, however, in a further communication to the Society, to show how the method enables the various damping-losses in an oscillating circuit to be determined separately by simple oscillographic means based upon observed deviations from the theory as outlined above.

# THE MEASUREMENT OF ELECTRICAL RESISTANCE IN TERMS OF A MUTUAL INDUCTANCE AND A PERIOD

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*ABSTRACT.* It is shown that if the ratio $\alpha/a$ of the radii of the concentric circles forming a simple inductometer is $0{\cdot}50607_8$, the mutual inductance is so accurately proportional to the angle through which the turning coil is displaced that the rising deviation from a linear law is less than 5 parts in a million at 7° of deflection. If the outer circle is replaced by two twin circles which are separated symmetrically with regard to the turning coil, the ratio $\alpha/a$ must be increased to bring about a similar approach to linearity.

If a heavy dynamometer is constructed on this principle, then, over the range indicated, the deflecting couple is strictly proportional to the current-products. Moreover if such a dynamometer be damped electrically, the damping-couple is accurately proportional to the velocity and, over a range of oscillation of 14° we have a remarkably close approach to true damped simple harmonic motion.

It is shown that the resistance of the oscillator circuit is very accurately given by the expression

$$R = \frac{CM}{c} \Big/ \left[ \frac{\lambda T}{\pi^2 + \lambda^2} - \frac{\lambda_0 T_0}{\pi^2 + \lambda_0^2} \right],$$

where $\lambda$ is the logarithmic decrement and $T$ the period on closed circuit of the oscillator when a current $C$ traverses the field coils, while $\lambda_0$ and $T_0$ correspond to open circuit and $M$ is the mutual inductance between the oscillating coil and the field coils when the former is in the position of steady deflection due to the passage through it of a steady current $c$, in conjunction with the current $C$ through the field coils. This expression eliminates quantities difficult to measure in Weber's method. Experimental tests are described. Table of Legendre and other functions, which facilitate the calculation of mutual inductances of the nature here considered, are appended.

## § 1. INTRODUCTION

A method of measuring electrical resistance absolutely was suggested by W. Weber*. In this method a magnet of moment $m$ is suspended within a coil and the period $T_0$ and logarithmic decrement $\lambda_0$ are observed when the circuit is open. On closure of the circuit induced currents assist the damping, and from a measurement of the new logarithmic decrement $\lambda$, together with other constants, the resistance of the coil can be calculated.

$m$

$T_0, \lambda_0$

$\lambda$

This historic method of damping has been carried out with various modifications by H. F. Weber, Dorn†; Wild‡; and F. Kohlrausch§.

* *Pogg. Ann.* **82**, 337 (1851). † *Wied. Ann.* **17**, 773 (1882); **36**, 22 and 398 (1889).
‡ *Mém. de l'Acad. des Sc. St Pétersbourg*, **32**, 2 (1884).
§ *Phil. Mag.* **47**, 302 and 342 1874); *Abh. d. bayr. Akad.* **16** (1888).

Unfortunately for the method, the basic formula for the resistance, namely

$$R = m^2 G^2 T_0 / 4I (\lambda - \lambda_0),$$

$R$ $G$ $I$

involves the square of a magnetic moment, the square of a galvanometer constant $G$ and a moment of inertia $I$ and in addition there are corrections for the arc of oscillation, for the length of the magnet and for the self-inductance of the coil. The attempts of other investigators to avoid these difficult quantities has resulted in the substitution of other quantities almost as difficult to measure accurately. Thus, in the opinion of Lord Rayleigh, Rowland and Glazebrook, the formula is sufficient to show that the method cannot compete with the other methods used for absolute measurements of resistance.

In 1930 the present writers* showed that, by the aid of a rotating coil magnetometer, the quantities $m^2 G^2 T_0 / 4I$ could be replaced by $\pi^2 MC / cT_0$, where $M$ is a mutual inductance and $C/c$ a current ratio.

In the present communication the magnet is replaced by a large, heavy coil of selected dimensions, the rotating coil magnetometer is no longer necessary, the swings are strictly isochronous, no length-correction is needed, the effect of self-

$T$

inductance can be eliminated by observation of $T$, the period on closed circuit, and the basic expression for the resistance of the oscillating coil is

$$R = \pi^2 MC / cT_0 (\lambda - \lambda_0).$$

Indeed the sole surviving measurements of Weber's historic method are those of period and logarithmic decrement, which, with a heavy system executing slow and true damped-simple-harmonic motion, can be measured with great accuracy. In order to attain these ends it is here shown that a suitable coil oscillating about a vertical axis may be so placed between two larger parallel field-coils that, over a range of some 7° on each side of the position of rest, the mutual inductance is so strictly proportional to the angle of displacement that the maximum deviation from a straight-line law is only a few parts in a million. Accordingly, when such a coil system is damped electrically, the damping-couple is accurately proportional to the angular velocity; and when it is used as a dynamometer, the deflecting couple is strictly proportional to the current-products.

As regards sensibility, it is practically immaterial whether the channel of the oscillating-coil-former is filled with copper wire of high or of low resistance. In either case it is equally convenient to standardize a low resistance.

## § 2. PRINCIPLE OF THE METHOD

A dynamometer is composed of two large field coils $A$, $B$, figure 3, of identical dimensions, together with a smaller heavy coil $C$ supported by a thick conducting torsion wire and fitted below with damping devices and a thin outlet spiral. The oscillating coil $C$ is wound upon a former in a channel of square section, and is of

$\alpha$

such dimensions that the ratio of the mean radius $\alpha$ of the turns composing it to

$a$

the mean radius $a$ of the turns of the field coils lies within the limits expressed by the relationship $0{\cdot}56 > \alpha/a > 0{\cdot}51$.

* *Proc. Phys. Soc.* **42**, 501 (1930).

The position of rest of $C$ is arranged to be that of zero mutual inductance with $A$ and $B$ when the latter are in series and conjunction. The mutual inductance must also be zero when $C$ is swinging and $A$ and $B$ in opposition. The distance of separation of $A$ and $B$ is then adjusted so that for deflections over the range of the scale the mutual inductance is proportional, within the accuracy of measurement, to the angle of displacement of $C$.

Observations of $T_0$, the period on open circuit, and $\lambda_0$, the corresponding logarithmic decrement, are then made with the aid of a telescope and circular scale. Next, with the oscillating coil short-circuited, a current $C$, between 1 and 2 amperes, is passed through the field coils, and the heavy logarithmic decrement $\lambda$ of the oscillating coil is measured. The current $C$ in the field coils being maintained, a small current $c$ (whose value relative to $C$ must be accurately known) is passed through the oscillating coil now critically damped, and a suitable scale-deflection is obtained. The oscillating coil is then clamped at this deflection, the currents are broken, and the mutual inductance $M$ between the field coils in conjunction and the oscillating coil is measured. By reversal of the current through the latter, a deflection on the other side of the zero position, and hence the mean mutual inductance corresponding to the current product $Cc$, is obtained. The total resistance of the closed oscillating coil circuit can then be found to a high order of accuracy from the simple expression

$T_0, \lambda_0$

$\lambda$

$C$

$c$

$M$

$$R = \frac{\pi^2 MC}{cT_0(\lambda - \lambda^3/2\pi^2 - \lambda_0)}.$$

## §3. THEORY OF THE METHOD

*Design of dynamometer coils so that over a small range the mutual inductance is very accurately proportional to the angle of rotation.* It was pointed out by Lord Rayleigh* as far back as 1886 that if the ratio of the mean diameters of the coils of a simple inductometer is about 0·55, the mutual inductance is very nearly proportional to the angle through which the turning coil is displaced from the zero position up to some 70° of range. Lord Rayleigh illustrated this by taking four terms of a slowly converging series for the case where the coils are concentric. The problem before us is to find the best value of the ratio $\alpha/a$ of the coil radii so that the law $M = k\theta$, where $k$ is a constant, shall hold with the highest precision over some few degrees of arc; and to investigate, further, how this ratio is affected when the rotating coil has its centre so situated as to lie at a distance $x$ from the centres of two twin coils symmetrically placed about it.

$\theta, k$

$x$

The mutual inductance between a circle $A$ and a smaller circle $C$ in the position shown in figure 1 is given† by

$$M_{AC} = 4\pi^2 s \sin^2\psi \sin^2\phi \sum_{n=1}^{n=\infty} \frac{1}{n(n+1)} P_n'(\cos\psi) P_n'(\cos\phi) P_n(\sin\theta) \left(\frac{s}{r}\right)^n \quad \ldots\ldots(1),$$

where $P_n$ is the Legendre function of the first kind, of order $n$, $P_n'$ its differential

* *Phil. Mag.* **22**, 469 (1886).

† Gray, *Absolute Measurements in Electricity and Magnetism*, pp. 201, 227.

$\alpha, s, r, \psi, \phi, \theta$ coefficient, and $n$ a positive integer, and where the symbols $s$, $r$, $\psi$, $\phi$, and $\theta$ are sufficiently defined by figure 1.

The presence of a second circle $B$ identical with $A$ and symmetrically placed with regard to $C$ simplifies the expression by removing even terms. We may then write for the mutual inductance between $C$ and the twin coils $A$ and $B$ in conjunction

$$M = Gq \sum_{n=1}^{n=\infty} \frac{2}{n(n+1)} \binom{s}{r}^{n-1} P_n'(\cos\psi)\, P_n'(\cos\phi)\, P_n(\sin\theta) \quad \ldots\ldots(2),$$

$q$ where $q = \pi\alpha^2$ and is the area of the small circle $C$, while $G = t.2\pi a^2 r^3$ and is the
$t$ galvanometer constant of $A$ and $B$ in conjunction at the origin $O$; $t$ is the total number of turns on $A$ and $B$ together, if the turns on each coil may be regarded as
$n$ coincident, and $n$ is now an odd positive integer.

As the complications attendant on the problem of multiple layers are usually best dealt with by Maxwell's method of subsequent correction rather than by direct integration of equation (2), we shall examine in the first place two cases of importance with simple circles.

*Case of two concentric circles.* When $A$ and $B$ are coincident and concentric with $C$ we have, figure 2, $r = a$, $s = \alpha$, $\phi = \psi = 90°$ and hence since

$$P_n'(\cos 90) = (-1)^{(n-1)/2}(1.3.5.7 \ldots n)/\{2.4.6 \ldots (n-1)\} \quad \ldots\ldots(3),$$

$$M = Gq \sum_{n=1}^{n=\infty} \frac{2}{n(n+1)} \frac{1^2.3^2.5^2.7^2 \ldots n^2}{2^2.4^2.6^2 \ldots (n-1)^2} P_n(\sin\theta)\, y^{(n-1)/2} \quad \ldots\ldots(4)$$

$y$ where $n$ is odd and $y = \alpha^2/a^2$.

By expanding $P_n(\sin\theta)$ in powers of $\sin\theta$ and these again in powers of $\theta$ we obtain the following useful expression

$$\begin{aligned}
\frac{M}{Gq} = {} & \theta \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} \\
& - \frac{\theta^3}{6} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} [1 + (n+2)(n-1)] \\
& + \frac{\theta^5}{120} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} [1 + 10(n+2)(n-1) \\
& \qquad + (n-3)(n-1)(n+4)(n+2)] \\
& - \frac{\theta^7}{5040} \sum_{n=1}^{n=\infty} (-y)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1^3.3^3.5^3 \ldots n^3}{2^3.4^3 \ldots (n-1)^3} \\
& \qquad \times [1 + 91(n+2)(n-1) + 35(n-3)(n-1)(n+4)(n+2) \\
& \qquad + (n-5)(n-3)(n-1)(n+6)(n+4)(n+2)] \\
& + \ldots\ldots\ldots \qquad \ldots\ldots(5),
\end{aligned}$$

where $n$ is odd and by the usual convention the first term ($n = 1$) under the summation sign is always unity.

When $y = 0$, that is when, in the limit, the rotating coil is of infinitely small area, the mutual inductance clearly follows a sine law deviating from a straight line as $\sin\theta$ deviates from $\theta$. When $y = 0{\cdot}256115$ or $\alpha = a = 0{\cdot}506078$ the term in $\theta^3$

vanishes, the coefficient being less than $1 \times 10^{-7}$, and on substituting this value in the other terms we obtain

$$M/Gq = 0{\cdot}87963773_0\,\theta - 0{\cdot}01687_9\,\theta^5 - 0{\cdot}00483\,\theta^7 \qquad \ldots\ldots(6).$$

This result, arrived at by summation of terms as far as $n = 31$, reveals a rising deviation from the straight-line law amounting to 4·2 parts in a million at 7° of deflection, 18 parts in a million at 10°, and 27 parts in a million at 11° of deflection.

The remarkable nature of this result is confirmed by direct calculation from the fundamental equation (4), and in table 1 we show the values of $M/Gq$ thus directly calculated for the angles 0° to 7° inclusive and when $\theta = 11°$, $y = 0{\cdot}2561$

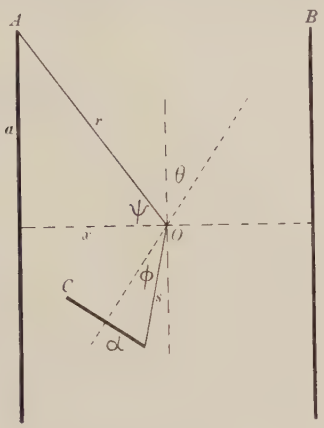


Fig. 1.

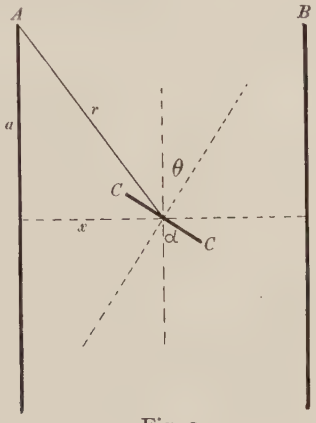


Fig. 2.

and $y = 0{\cdot}2567$. This calculation necessitates a knowledge of $P_n(\sin\theta)$ for the angles involved up to $n = 21$. When $y = 0{\cdot}2567$, and is therefore slightly larger than the limiting solution, the deviation is distributed, the maximum deviation over the range 0° to 7° never exceeding 0·75 part in a million.

Table 1. Variation of mutual inductance with angle of deflection for special cases of concentric circles

| Angle (°) | Actual value of $M/Gq$ $\alpha^2/a^2 = 0{\cdot}2561$ | Value of $M/Gq$ if linear law be assumed | Difference $\times 10^8$ | Actual value of $M/Gq$ $\alpha^2/a^2 = 0{\cdot}2567$ | Difference from linear-law value $\times 10^8$ |
|---|---|---|---|---|---|
| 0 | 0·00000000 | 0·00000000 | 0·0 | 0·00000000 | 0·0 |
| 1 | $0{\cdot}01535267_8$ | $0{\cdot}01535267_8$ | 0·0 | $0{\cdot}01534855_8$ | 0·0 |
| 2 | $0{\cdot}03070535_6$ | $0{\cdot}03070535_6$ | 0·0 | $0{\cdot}03069712_2$ | + 0·6 |
| 3 | $0{\cdot}04605802_7$ | $0{\cdot}04605803_4$ | − 0·7 | $0{\cdot}04604569_5$ | + 2·1 |
| 4 | $0{\cdot}06141068_1$ | $0{\cdot}06141071_2$ | − 3·1 | $0{\cdot}06139427_5$ | + 4·3 |
| 5 | $0{\cdot}07676330_1$ | $0{\cdot}07676339_0$ | − 8·9 | $0{\cdot}07674284_6$ | + 5·6 |
| 6 | $0{\cdot}09211584_8$ | $0{\cdot}09211606_8$ | − 22·0 | $0{\cdot}09209138_4$ | + 3·6 |
| 7 | $0{\cdot}10746827_3$ | $0{\cdot}10746874_6$ | − 47·3 | $0{\cdot}10743984_5$ | − 6·1 |
| 11 | $0{\cdot}16887496_3$ | $0{\cdot}16887945_8$ | − 449·5 | $0{\cdot}16883127_9$ | − 285·8 |

It might well be questioned whether a coil could in practice be constructed near enough to the theoretical specification to yield anything like the best possible

linearity between $M$ and $\theta$. It should be pointed out at once therefore that this difficulty is largely overcome by the fact that if the oscillating coil is somewhat larger than the value demanded by the concentric position, twin field-coils may be placed on either side of it and separated outwards till linearity is attained; for it will be seen below that as the coils are separated a larger and larger value of $y$ is required—a fact, moreover, which is likely to be of great importance when correction is made for the action of multiple layers.

*Case of circle with centre symmetrically on the axis of two larger equal parallel coils.* This case, shown in figure 2, is derived from the more general case of figure 1 by putting $\phi$ equal to 90° and $s$ equal to $\alpha$. This case is of practical importance and equation (2) becomes

$$\frac{M}{Gq} = \sum_{n=1}^{n=\infty} (-z)^{(n-1)/2} \frac{2}{n(n+1)} \frac{1 \cdot 3 \cdot 5 \ldots n}{2 \cdot 4 \ldots (n-1)} P_n'(\cos\psi) P_n(\sin\theta) \qquad \ldots\ldots(7),$$

$n, z$ where $n$ is an odd positive integer and $z = \alpha^2/r^2$.

By expansion in powers of $\theta$ this gives the expression

$$\begin{aligned}\frac{M}{Gq} = {} & \theta \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2 \cdot 3^2 \cdot 5^2 \ldots n^2}{2^2 \cdot 4^2 \ldots (n-1)^2} \\ & - \frac{\theta^3}{6} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2 \cdot 3^2 \cdot 5^2 \ldots n^2}{2^2 \cdot 4^2 \ldots (n-1)^2} [1 + (n+2)(n-1)] \\ & + \frac{\theta^5}{120} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2 \cdot 3^2 \cdot 5^2 \ldots n^2}{2^2 \cdot 4^2 \ldots (n-1)^2} \\ & \qquad \times [1 + 10(n+2)(n-1) + (n+4)(n+2)(n-3)(n-1)] \\ & - \frac{\theta^7}{5040} \sum_{n=1}^{n=\infty} z^{(n-1)/2} P_n'(\cos\psi) \frac{2}{n(n+1)} \frac{1^2 \cdot 3^2 \cdot 5^2 \ldots n^2}{2^2 \cdot 4^2 \ldots (n-1)^2} \\ & \qquad \times [1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-3)(n-1) \\ & \qquad\qquad + (n-6)(n-4)(n-2)(n-5)(n-3)(n-1)] \\ & + \ldots\ldots\ldots \qquad \ldots\ldots(8).\end{aligned}$$

A preliminary investigation of this expression up to the fourth term clearly reveals that as $x/a$ or $\cot\psi$ increases continuously the term in $\theta^3$ vanishes for larger and larger values of $z$, a second and higher root also appearing; $y$ or $\alpha^2/a^2$ attains a value of about 0·4 at $x^2/a^2 = 0{\cdot}05$, and the terms in $\theta^5$ and $\theta^7$ are still very small. Above this the roots approach and merge, the term in $\theta^3$ becoming a minimum and almost vanishing for very slowly diminishing values of $z$.

As our actual experiments were performed with $\psi$ equal to 79° approximately, we have worked out this case completely, using accurate values of $P_n'(\cos 79)$ up to $n = 31$.

When $\psi = 79°$, $x^2/a^2 = 0{\cdot}037784$, $x/a = 0{\cdot}19438$, the term in $\theta^3$ vanishes if $z = 0{\cdot}2874$*, i.e. when $y = \alpha^2/a^2 = 0{\cdot}29826$ or $\alpha/a = 0{\cdot}5461$. On substituting this value in the other terms of equation (8), we obtain

$$M/Gq = 0{\cdot}88498462\,\theta - 0{\cdot}02030\,\theta^5 - 0{\cdot}0063\,\theta^7 \qquad \ldots\ldots(9).$$

* The root is possibly nearer 0·2873. When $z = 0{\cdot}2874$ the residual term in $\theta^3$ is $+0{\cdot}0000245\theta^3$.

This result reveals a deviation of 5·1 parts in a million from the straight line law at $\theta = 7°$, of 21 parts in a million at 10° and of 31 parts in a million at 11°. It is confirmed by direct calculation from the fundamental equation (7). In table 2 we give the values of $M/Gq$ thus directly calculated for the angles 0° to 7° inclusive as well as for 11°; this direct calculation needs an accurate knowledge of $P_n(\sin\theta)$ for the angles involved up to $P_{21}(\sin\theta)$. It should be noted that in evaluating the first one or two terms, direct multiplication is necessary, seven-figure logarithms not being sufficiently accurate.

Table 2. Variation of mutual inductance with angle of deflection for circle between two larger equal circles. (Ratio of radii $\alpha/a = 0{\cdot}5461$; measure of displacement $x/a = 0{\cdot}19438$)

| Angle (°) | Actual value of $M/Gq$ | Value of $M/Gq$ on assumption of a linear law | Difference $\times 10^8$ |
|---|---|---|---|
| 0 | 0·00000000 | 0·00000000 | 0·0 |
| 1 | 0·01544589$_5$ | 0·01544589$_5$ | 0·0 |
| 2 | 0·03089179$_0$ | 0·03089179$_0$ | 0·0 |
| 3 | 0·04633768$_0$ | 0·04633768$_5$ | − 0·5 |
| 4 | 0·06178355$_5$ | 0·06178358$_0$ | − 2·5 |
| 5 | 0·07722939$_0$ | 0·07722947$_5$ | − 8·5 |
| 6 | 0·09267514$_3$ | 0·09267537$_0$ | − 22·7 |
| 7 | 0·10812075$_7$ | 0·10812126$_5$ | − 50·8 |
| 11 | 0·16989967$_0$ | 0·16990484$_5$ | − 517·5 |

*Effect of coils possessing multiple layers.* Though we are unable to offer a satisfactory mathematical treatment of this problem, we have derived useful information by making use of a principle due to Maxwell. If a coil of rectangular cross-section has axial breadth $2b$ and radial depth $2d$, and $P_0$ is any term in the expression of action of any kind at a point, due to the central circular filament of radius $\alpha$, then, to a first order, the average term $\bar{P}$ for the action of the entire coil is given by

$$\bar{P} = P_0 + \frac{(b^2 + d^2)}{6}\frac{\partial^2 P_0}{\partial a^2} \qquad \ldots\ldots(10).$$

It is much easier to apply this equation to the oscillating coil than to the field-coils and as a result we aim at so constructing the coil former that $1 > d/b > 0{\cdot}9$, under which circumstances, if $d$ and $b$ are small compared with the radius $\alpha$, the disturbing effect is very slight. If the oscillating coil is unduly lengthened so as to approach the nature of a solenoid, its radius $\alpha$ must be increased to maintain proportionality between mutual inductance and deflection. As the primary effect of multiple layers is to alter the effective radii of coils, it is highly probable that the effect may in the main be compensated by adjustment of the distance between the field-coils. At the moment we are submitting problems of this nature to direct experimental test.

*Determination of the resistance by observation of the damping of the oscillating coil of a dynamometer fulfilling the law $M = k\theta$.* Consider a coil suspended symmetrically between twin field coils so adjusted that over 7° on either side of the

position of rest (which is that of zero mutual inductance) the mutual inductance is very accurately proportional to the angle of deflection $\theta$. When the field-coils are carrying a current $C$ and the other coil a current $c$, the mutual energy of the system is $Cck\theta$, and the couple tending to increase $\theta$ is $Cck$, being independent of the angle. The action of the earth's magnetic field not only is extremely minute but is completely neutralized in our experiments by large square coils arranged in the Helmholtz manner, and hence it need not be considered further.

Now if while a current $C$ traverses the field-coils the oscillating-coil circuit is closed through a resistance, a current $c_1$ which varies with time will be induced in the moving coil and give rise to electrical damping. As the flux $F$ in the position $\theta$ is given by

$$F = Ck\theta + Lc_1 \qquad \text{.....(11),}$$

where $L$ is the self-inductance of the moving coil, we have for the induced current

$$c_1 = (-Ck\dot{\theta} - L\dot{c}_1)/R \qquad \text{.....(12),}$$

where $R$ is the total resistance of the closed oscillator circuit.

Since the effect of self-inductance is small, we may take $\dot{c}_1$ in the $L\dot{c}_1$ term as the value of $\dot{c}_1$ when inductance is neglected, namely:

$$-Ck\dot{\theta}/R,$$

and hence

$$c_1 = -Ck\dot{\theta}/R + LCk\ddot{\theta}/R^2 \qquad \text{.....(13).}$$

Thus the deflecting couple, tending to increase $\theta$, which is produced by the current $C$ in the field coils and the induced current $c_1$ in the oscillating coil, is given algebraically by

$$Cc_1k = -C^2k^2\dot{\theta}/R + LC^2k^2\ddot{\theta}/R^2 \qquad \text{.....(14),}$$

so that on equating the resultant torque, tending to increase $\theta$, to the product of moment of inertia and angular acceleration, we have

$$(I - LC^2k^2/R^2)\,\ddot{\theta} + (p + C^2k^2/R)\,\dot{\theta} + u\theta = 0 \qquad \text{.....(15),}$$

where $p$ is the coefficient of air damping and $u$ the torsional restoring couple per radian, so that, without approximation,

$$C^2k^2/R = 4I\,(\lambda/T - \lambda_0/T_0) - (4\lambda/T)\,LC^2k^2/R^2 \qquad \text{.....(16),}$$

where $\lambda_0$, $T_0$ are the logarithmic decrement and period respectively on open circuit and $\lambda$, $T$ the corresponding quantities when the coil is closed through the total resistance $R$. Since the effect of self-inductance is very small, this quadratic in $R$ is conveniently written without approximation in the form

$$R = \frac{C^2k^2}{4I}\,\frac{1}{\lambda/T - \lambda_0/T_0}\left[1 + \frac{4\lambda}{T}\,\frac{L}{R}\right] \qquad \text{.....(17).}$$

Now since in oscillation on open circuit

$$u/4I = (\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(18),}$$

and since on deflection by a steady current $c$ through the oscillating coil while the current $C$ traverses the field coils

$$Cck = u\theta \qquad \text{.....(19),}$$

where

$$k\theta = M, \qquad \text{.....(20),}$$

$M$ being the measured mutual inductance when the oscillating coil is damped in the position which it takes up when a steady deflection is produced by the combined currents $C$ and $c$, we have, on multiplying equations (18), (19) and (20) together, $M$

$$C^2k^2/4I = M\,(C/c)\,(\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(21),}$$

and on making this substitution for $C^2k^2/4I$ in equation (17)

$$R = M\frac{C}{c}\,\frac{\pi^2 + \lambda_0^2}{T_0^2\,(\lambda/T - \lambda_0/T_0)}\left[1 + \frac{4\lambda}{T}\,\frac{L}{R}\right] \qquad \text{.....(22),}$$

or very approximately

$$R = M\frac{C}{c}\,\frac{\pi^2 + \lambda_0^2}{T_0^2\,(\lambda/T - \lambda_0/T_0)} + \frac{4\lambda L}{T} \qquad \text{.....(23).}$$

As, within the accuracy of our experimental tests, $4\lambda L/RT$ is negligible compared with unity, the effect of self-inductance may be neglected in our case and accordingly

$$(\pi^2 + \lambda^2)/T^2 = (\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{.....(24);}$$

hence $T$ can be calculated from $T_0$, and indeed with ample accuracy

$$T = T_0\,(1 + \beta) \qquad \text{.....(25),}$$

where

$$\beta = (\lambda^2 - \lambda_0^2)/2\pi^2 \qquad \text{.....(26),} \qquad \beta$$

and thus

$$R = M\frac{C}{c}\,\frac{\pi^2 + \lambda_0^2}{T_0\,[\lambda\,(1 - \beta) - \lambda_0]} \qquad \text{.....(27);}$$

or in our experiments, on account of the smallness of $L$ and $\lambda_0$,

$$R = M\,(C/c)\,\pi^2/T_0\,(\lambda - \lambda^3/2\pi^2 - \lambda_0) \qquad \text{.....(28),}$$

with an accuracy never less than one part in ten thousand.

*Elimination of the primary effect of self-inductance.* In the solution of the fundamental differential equation (15) we have, before introducing the special elimination of $C^2k^2/4I$, followed the usual lines associated with Weber's method of damping, by treating the effect of self-inductance as a very small addition term—a procedure most useful in showing the smallness and estimating the order of the effect.

It may, however, ultimately be desirable to use larger coils in which the ratio $L/R$ of the oscillator circuit is increased, and we here show that the effect of self-inductance may be completely eliminated from equation (15), which recognizes the primary effect of such inductive action in diminishing the effective moment of inertia of the system. From equation (15) we have, under closed-circuit conditions,

$$u/(I - LC^2k^2/R^2) = 4\,(\pi^2 + \lambda^2)/T^2 \qquad \text{.....(29),}$$

and combining this with equation (18) for open circuit we find

$$LC^2k^2/R^2I = 1 - (\pi^2 + \lambda_0^2)\,T^2/(\pi^2 + \lambda^2)\,T_0^2 \qquad \text{.....(30),}$$

and so on, making this substitution in equation (16),

$$\frac{C^2k^2}{R} = 4I\left[\frac{\lambda}{T}\,\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda^2}\,\frac{T^2}{T_0^2} - \frac{\lambda_0}{T_0}\right] \qquad \text{.....(31);}$$

whence, on making the special substitution of equation (21), we obtain

$$R = MC/c\,\{\lambda T/(\pi^2 + \lambda^2) - \lambda_0\,T_0/(\pi^2 + \lambda_0^2)\} \qquad \text{.....(32),}$$

an accurate expression eliminating the primary inertia effect of self-inductance and derived without approximation from equations (15) and (21).

This equation could be used directly, to the fullest advantage of the method, if the period $T$ (as the number of swings is limited) were taken photographically by a running film on which an accurate time trace was simultaneously recorded*.

If $T$ cannot be observed experimentally with sufficient accuracy and the effect of self-inductance is not negligible but small (i.e. $4\lambda L\ RT_0$ is small compared with unity), $L/R$ must be estimated by measuring $L$ and using the value of $R$ obtained by neglecting inductance. From equations (30) and (16) or (31) we then have, as a good approximation,

$$(\pi^2 + \lambda_0^2)\ T^2/(\pi^2 + \lambda^2)\ T_0^2 = 1 - 4L\ (\lambda - \lambda_0)/RT_0 \qquad \text{.....(33),}$$

or

$$T = T_0\ (1 + \beta - \gamma) \qquad \text{.....(34),}$$

where $\beta = (\lambda^2 - \lambda_0^2)/2\pi^2$ as before (26) and

$$\gamma = 2L\ (\lambda - \lambda_0)/RT_0 \qquad \text{.....(35),}$$

whence from equation (32)

$$R = M\ (C/c)\ (\pi^2 + \lambda_0^2)/T_0\ [\lambda\ (1 - \beta - \gamma) - \lambda_0] \qquad \text{.....(36),}$$

or (very closely, as $\lambda_0$ is very small)

$$R = M\frac{C}{c}\frac{\pi^2}{T_0\ (\lambda - \lambda^3/2\pi^2 - \lambda_0)} + \frac{2L}{T_0}\ (\lambda - \lambda_0) \qquad \text{.....(37).}$$

These last equations emphasize the care that must be taken in estimating $T$ if the inductance term is too large to be neglected.

It is unsound, for example, in using equation (23) to estimate $T$ as in equations (24) and (25), neglecting inductance, and then to add on to the value of $R$ the quantity $4\lambda L/T$ as a correction for inductance.

*Separation of the dynamical and statical experiments.* The logarithmic decrements $\lambda$ and $\lambda_0$ are taken under periods defined by $T$ and $T_0$. We have assumed that when the deflection experiments are performed the control is unchanged and that the natural period on removal of the damping vanes is still $T_0$. If, owing to change of temperature, the natural period taken before and after the deflection experiments differs from $T_0$ and has a value $T_1$, equation (21) should be replaced by

$$\frac{C^2k^2}{4I} = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_1^2} = M\frac{C}{c}\frac{\pi^2 + \lambda_0^2}{T_0^2}\frac{T_0^2}{T_1^2} \qquad \text{.....(38),}$$

and accordingly any expression for $R$ must be multiplied by the factor $T_0^2/T_1^2$. Thus, since inductance is negligible in our experiments, our working formula, as derived from equation (28), is

$$R = M\frac{C}{c}\frac{\pi^2}{(\lambda - \lambda^3/2\pi^2 - \lambda_0)}\frac{T_0}{T_1^2}, \qquad \text{.....(39),}$$

though in practice the difference between $T_0$ and $T_1$ is normally only one or two parts in ten thousand.

* Cf. *Dictionary of Applied Physics*, **2**, 235.

*The sensibility in relation to the dimensions and windings of the apparatus.* The fundamental feature of the important problem of sensibility lies in the necessity, on changing from mechanical to electrical damping, of securing a marked and accurately measurable increase of logarithmic decrement. Whatever the moment of inertia of the oscillating coil may be, it is desirable to maintain a period of about 16 seconds by suitable choice of the length and diameter of the suspending wire—at all events if logarithmic decrements are measured by direct eye-observation. Regarding then $T_0$ as fixed, we examine the sensibility by means of the expression

$$(\lambda - \lambda_0)/T_0 = C^2k^2/4IR = C^2\,(0{\cdot}88Gq)^2/4IR \qquad \text{.....(40).}$$

Now if the channel of the oscillating coil is of volume $V$ and mean radius $\alpha$ and if $\sigma$ is the resistivity and $l$ the length of wire filling it, then whatever the gauge of wire chosen, since the resistance of the suspension may be regarded as negligible, we have $V, \alpha$ $\sigma, l$

$$R = \sigma l^2/V \qquad \text{.....(41)}$$

and

$$q = \alpha l/2 \qquad \text{.....(42),}$$

and hence

$$(\lambda - \lambda_0)/T_0 = 0{\cdot}048\ C^2G^2\alpha^2V/I\sigma \qquad \text{.....(43),}$$

which reveals that the sensibility is independent of the diameter and length of the wire.

If the former of the oscillating coil is of negligible moment of inertia in comparison with the wire it contains, $\alpha^2V$ is proportional to $I$. Thus the sensibility is proportional to $CG^2$ and inversely proportional to the resistivity of the material with which the oscillating coil is wound. Now $G$ is dependent on the dimensions of the field coils, which have double the radius of the oscillating coil and if the field coils are made larger in all proportions and wound with a fixed gauge of wire, $G$ will increase with $a$, the mean field coil radius. $a$

Thus the larger the entire apparatus the greater is the sensibility, the surplus of which may well be utilized in reducing $C$, the current through the field coils, or the channels of the field coils or possibly in substituting manganin for copper in the oscillating coil. Other advantages of working with a large apparatus are that $\lambda_0$ becomes very small and constant, that a thick fatigueless suspension may be used, that the stability is excellent and that ample clearance is left between the field-coil formers and the sphere surrounding the oscillating coil.

## § 4. THE APPARATUS AND METHOD OF EXPERIMENTING

Under difficult circumstances, due to insufficient space and isolation and to the vibration of the building, we have tested experimentally the general theory of the method with the apparatus shown in figure 3.

The field coils $A$, $B$ were constructed of dexonite specially cast by the Dexine Co. Ltd., the channels being of radial depth and axial breadth 3·6 cm. and the mean diameter of the winding 31·0 cm. Each channel was filled with 504 turns of double-silk-covered copper wire (s.w.g. 16), and the galvanometer-constant $G$ with the coils touching and joined in series and conjunction was 368 cm.$^{-1}$ at the origin

of symmetry. The coils could be separated and firmly locked and were capable of rotation on the carrier *D* about a vertical axis. They were set with the plane of the windings in the magnetic meridian.

Several coils were used as the oscillating coil *C*. Each was of mean channel-diameter about 17 cm. but of varying dimensions which are given in the next section,

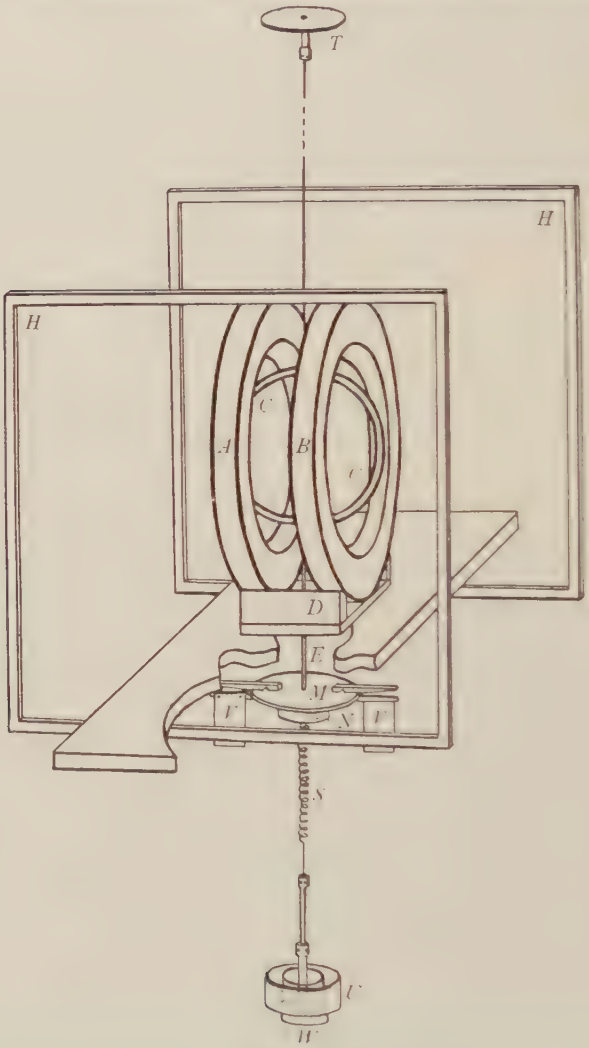


Fig. 3.

and was wound on a wooden former, the ends of the windings being brought to brass cross-pieces 180° apart. It was encased in a papier-mâché sphere (a globe of the world was used for this purpose) which not only diminished and smoothed the air damping but also enabled the moving system to be neatly clamped, without jolt or displacement, by means of wooden jaws capable of adjustment and controlled by a link work slider.

Current is led into the oscillating coil via the torsion head $T$ and suspension wire, and passes out via a vertical brass rod $E$, finishing in a spiral $S$ attached to a firm base $W$. The connecting leads to $T$ and $W$, along the ceiling and floor respectively, are parallel to the axis of the field coils.

Attached to the rod $E$ is a horizontal disc $M$ to which, when necessary, can easily be clipped two vanes of aluminium, $VV$, dipping into troughs of liquid paraffin and giving critical damping to the rotating system. This liquid was most satisfactory and vastly superior to glycerin, which is not only hygroscopic but, owing to the heaping up of the liquid by the vanes in rotation, produces a kind of fatigue. Under the disc $M$ is a concentric copper cylinder $N$ which dips into a small annular glass vessel $U$ (shown in figure 3 as resting on the base $W$) surrounding the lower suspension and filled with xylene. The effect of this is to damp out lateral movements of the rotating system due to the vibration of the room, and to ensure smoothness of rotation without producing any appreciable damping of the torsional oscillations. A device of this nature was essential for the accurate measurement of logarithmic decrements.

The weight of the oscillating coil and its accessories was often as large as 2000 gm., and the suspension was a metre of German-silver wire, s.w.g. 22. This was quite fatigueless. Earlier suspensions of silicon-bronze and cadmium copper showed slight traces of fatigue, and our attempts to obtain elinvar were unsuccessful. Thick wires show far less fatigue than thin ones, but no other wire below gauge 18, examined in a careful search, behaved with the striking perfection of this German silver. The outlet spiral in the earlier experiments was of silicon bronze, s.w.g. 30, and in later experiments of copper wire, s.w.g. 36.

A plane mirror was attached to the rod socket where it emerged from the globe at the bottom of the coil. This mirror, in conjunction with a telescope and well illuminated scale, allowed deflections, period and logarithmic decrements to be measured. The scale was circular, its radius of curvature was 198·5 cm. and its length 1 m., and the zero was in the middle. Readings of steady deflections could be measured to 0·01 cm.

The earth's horizontal field at right angles to the axis of the field coils was neutralized by two large rectangular coils, figure 3, of sides 49 cm. by 55 cm., $HH$, placed in the Helmholtz position and worked from a single accumulator. The neutralization of the earth's field by the flux of these earth coils was finally checked with all *in situ* by timing the swings of the oscillating coil when it carried (*a*) no current, (*b*) a current of 1·5 A, (*c*) a similar current in the contrary direction; equality of all these periods being secured by adjustment of the earth-coil current. For this purpose the lower spiral was removed and replaced by a pin dipping into mercury.

The entire dynamometer was enclosed in a case and the suspensions and damping appliances were shrouded so that the entire system was free from draughts. The leads from the oscillating coil were brought through a resistance box to opposite quadrants of a good key situated close to the telescope, where either a short circuit could be effected or connection could be made through potential leads to

a potentiometer in either direction, or through current leads to an accumulator circuit or alternatively a mutual inductometer. The leads from the field coils were led well away to keys which enabled them to be joined in conjunction or opposition, and thence in either direction to the mutual inductometer or to an accumulator circuit.

Before an experiment the torsion head was so adjusted that the oscillating coil in its position of rest, with no torsion in either top or bottom suspension, had zero mutual inductance with the field coils. The mirror was then adjusted so that the reading in the telescope was very near the middle of the scale. Furthermore the symmetry was so adjusted that when the field coils were in opposition the mutual inductance was very closely zero over the entire scale-range of oscillation. Readings were then taken, at every 10 cm. of the scale, of the mutual inductance between the oscillating coil and the combined field coils. Inductances and deflections of nearly equal magnitudes on opposite sides of the zero were subtracted algebraically to eliminate any slight displacement of the scale and any irreversible inductance. While the linearity between mutual inductance and displacement were thus being tested the deflections were always produced electrically.

The mutual inductometer was of the latest Campbell pattern of range 0–11,000 $\mu$H with $\frac{1}{5}$ factor switch for the scale, and the mutual inductances were measured in the first place by Hartshorn's modification* of Felici's method using a.c. of low frequency from a valve oscillator, the readings being standardized by d.c. and ballistic-galvanometer observation. Later we often worked directly by d.c., which, to our surprise, we found to be sensitive to about 0·2 $\mu$H at mean range. By suitable switches we could change immediately from a.c. to d.c. The inductometer, dynamometer and valve oscillator were all well separated.

A typical day's experiment is illustrated by the following order of observation: (*a*) The zero and symmetry are tested and if need be corrected and the $\mu$H scale-division checked by taking readings near the ends of the scale and at about 20 cm. on each side of the zero. (*b*) A period $T_0$ is taken and $\lambda_0$, the air-damping decrement, observed at the same time. (*c*) A current $C$, usually between 1·25 and 1·55 A., is passed through the field coils and maintained very steady. The current value is read off a standard 0·01 ohm on a Cambridge thermoelectric potentiometer reading from 0 to 90 mV. While one observer watches the current the other deflects the oscillating coil electrically, short-circuits it, and takes readings of the decrement. $\lambda$ is observed again almost immediately. Directly this reading is completed, the resistance $R$ of the oscillating-coil circuit is compared on the potentiometer with a 10- or 20-ohm resistance derived from the ratio arms of a good Post Office box. More decrements and more resistance comparisons are then taken, and the whole process is then repeated with different values of $C$. (*d*) A period $T_0$ and an air-logarithmic-decrement $\lambda_0$ are again taken. (*e*) The damping vanes are now most carefully clipped to the carrier disc and the troughs containing liquid paraffin are arranged to give critical damping. With a current $C$ (of the same magnitude as that used for the electrical-damping experiments) traversing the field coils, a

* *J. of Sci. Inst.* **2**, 145 (1925).

current $c$ is passed through the oscillating coil to produce a large steady deflection, which is noted. The current $c$ is then reversed and the deflection on the other side is read. This process is repeated with different values of $c$, and then again with other values of $C$ in the field coils, just as in the decrement experiments. As the $\mu$H. per scale-division is accurately determined, values of $M/Cc$ are easily calculated. The current $c$, of 1 to 5 mA, is measured on the Cambridge potentiometer by drawing the e.m.f. off the very same 10- or 20-ohm resistance that was used for comparison with $R$ between the decrements in process ($c$) above. ($f$) The damping vanes are removed and a period is taken which, averaged with the period in ($d$) above, gives $T_1$ of equation (39).

It will readily be seen that by measuring the current $c$ off the very resistance $R_s$ used for checking $R$, we throw the whole of the onus of the agreement of $R$, as measured by our method and by comparison with $R_s$, on the value of the standard 0·01 ohm, which was correct to one part in two thousand.

Experiments were only useful if performed in the quiet of the night or a week-end, for logarithmic decrements were completely spoilt if people were moving on the floor above us.

## § 5. EXPERIMENTAL TESTS

Preliminary experiments to test the possibilities of the method were made with an oscillating coil $C_1$ of about 85 turns of double-silk-covered copper wire of s.w.g. 22 supported by a silicon bronze suspension of s.w.g. 26. This coil, of mean radius $a = 8{\cdot}6$ cm., yielded a remarkably linear scale-calibration of 17·24 $\mu$H./cm., and a value at 14° C. for its resistance $R$ of 2·51 ohms, in close agreement with a Post Office box check. As a result of these initial tests, the equipment was at once improved by introducing a telescope and scale, a potentiometer system for the rapid checking of $R$, the Hartshorn impurity device with d.c. calibration, and it was decided to prepare coils of greater weight and to investigate the elastic properties of thick suspensions.

A coil $C_2$ was accordingly prepared of the following dimensions: depth of channel = breadth of channel, 2 cm.; internal radius of channel in coil former, 7·68 cm.; mean radius of winding $a$, 8·7 cm.; number of turns, 230½ of double-silk-covered copper wire of s.w.g. 18; self-inductance $L$, 13680 $\mu$H. This coil was suspended by fatigueless German silver wire, s.w.g. 22. In tables 3 and 4 we

Table 3. Microhenries and scale-displacement for oscillating coil $C_2$

| Scale-displacement (cm.) | Microhenries | $\mu$H./cm. from $n$th and $(n+3)$rd readings |
|---|---|---|
| 98·72 | 4383·4 | — |
| 80·50 | 3573·1 | — |
| 60·68 | 2694·4 | — |
| 40·25 | 1786·2 | 44·41$_9$ |
| 20·70 | 917·2 | 44·41$_3$ |
| 0·00 | − 0·9 | 44·41$_8$ |

Mean result 44·41$_7$ $\mu$H/cm.

illustrate the scale-calibration and the constancy of the ratio $M\,Cc$; and in tables 5 and 6, determinations of the resistance of the coil and its attachments and leads together with the values obtained by potentiometer comparison.

Table 4. Values of $M/Cc$ for oscillating coil $C_2$

$\mu$H/cm. = 44·41$_7$, $T_1$ = 15,680 sec.

| Field-coil current $C$ (A.) | Oscillating-coil current $c$ (A.) | Deflection due to reversal from $+c$ to $-c$ (cm.) | $M/Cc$ ($\mu$H./A.$^2$) |
|---|---|---|---|
| 2·4500 | 0·0035760 | 98·07 | 248595 |
| 2·2500 | 0·0035580 | 89·62 | 248620 |
| 2·0500 | 0·0042855 | 98·35 | 248621 |
| 1·7500 | 0·0050269 | 98·48 | 248616 |
| 1·5500 | 0·0050261 | 87·24 | 248698 |
| 1·3600 | 0·0058854 | 89·61$_5$ | 248648 |

Mean result 248630 $\mu$H./A.$^2$.

Table 5. Determination of resistance on April 24, 1931

$M/Cc = 2{\cdot}486_3 \times 10^8$ cm./A.$^2$; $T_1$ = 15·680 sec.; $T_0$ = 15·682 sec.; $\lambda_0$ (common) = 0·00096.

| $C$ | $\lambda$ (common) | $\lambda(1-\beta)-\lambda_0$ | $R \times 10^9$ (c.g.s.) | Resistance by comparison (ohms) | Remarks |
|---|---|---|---|---|---|
| — | — | — | — | 2·8459 | |
| 1·7500 | 0·07416 | 0·1683$_0$ | 2·848$_2$ | — | Resistance rising |
| 1·7500 | 0·07390 | 0·1677$_0$ | 2·858$_4$ | — | *C* too strong |
| — | — | — | — | 2·8574 | |
| 1·5500 | 0·05819 | | | | |
| 1·5500 | 0·05820 | 0·1316$_8$ | 2·855$_8$ | — | Resistance steady |
| 1·5500 | 0·05819 | | | | |
| — | — | — | — | 2·8534 | |
| 1·3600 | 0·04502 | 0·1013$_9$ | 2·855$_4$ | — | |
| — | — | — | — | 2·8531 | |

Another coil $C_3$ was prepared on a former of the same dimensions as that of $C_2$, wound with about 600 turns of double-silk-covered copper wire, s.w.g. 22, the mean radius of winding $x$ being 8·0 cm., the resistance without suspensions 14·9 ohms at 20° C., and the self-inductance $L$ 104700 $\mu$H. The suspension was of German silver, s.w.g. 22, and additional resistance was added from a non-inductive box to make the resistance approximately 20 ohms and hence more quickly and accurately comparable with the 20-ohm ratio-arm standard.

In table 7 we show how the microhenries varied with the scale deflection. The result clearly reveals that for this multiple layered coil $x/a$ was rather small: the field coils were as close as possible.

Table 6. Determination of resistance of oscillating coil $C_2$ on May 3, 1931

$M/Cc = 2{\cdot}487_2 \times 10^8$ cm./A.$^2$; $T_1 = 15{\cdot}702$ sec.; $T_0 = 15{\cdot}704$ sec.;
$\lambda_0$ (common) $= 0{\cdot}000945$.

| $C$ | $\lambda$ (common) | $\lambda(1-\beta)-\lambda_0$ | $R \times 10^9$ (c.g.s.) | Resistance by comparison (ohms) | Remarks |
|---|---|---|---|---|---|
| — | — | — | — | 2·8616 | |
| 1·4500 | 0·05082 | $0{\cdot}11147$ | $2{\cdot}864_3$ | — | Resistance rising |
| 1·3800 | $0{\cdot}046044$ | $0{\cdot}10378$ | $2{\cdot}869_2$ | — | Insufficient comparisons taken |
| — | — | — | — | 2·8671 | |
| 1·3500 | 0·04414 | 0·09941 | $2{\cdot}866_6$ | — | |
| — | — | — | — | 2·8656 | |
| 1·2500 | 0·03794 | 0·08515 | $2{\cdot}869_1$ | — | Extra resistance added to oscillating-coil circuit |
| — | — | — | — | 5·8580 | |
| 1·4500 | 0·02533 | 0·05614 | $5{\cdot}855_7$ | — | Resistance very steady |
| 1·4500 | 0·02533 | | | | |
| 1·4500 | 0·02533 | | | | |

Table 7. Microhenries and scale-displacement for coil $C_3$

| Scale-displacement (cm.) | Microhenries | $\mu$H./cm. from $n$th and $(n+5)$th readings |
|---|---|---|
| 99·68 | 12300·5 | — |
| 90·20 | 11130·5 | — |
| 80·36 | 9917·0 | — |
| 68·80 | 8489·6 | — |
| 63·60 | 7849·5 | — |
| 49·75 | 6142·5 | $123{\cdot}33_3$ |
| 37·23 | 4596·5 | $123{\cdot}35_3$ |
| 24·20 | 2988·2 | $123{\cdot}37_6$ |
| 9·81 | 1211·3 | $123{\cdot}38_2$ |
| 0·00 | 0·0 | $123{\cdot}42_0$ |

We give in tables 8, 9 and 10 the complete data of a long experiment performed on June 21 with the coil $C_3$ after careful resetting for zero and symmetry.

Table 8. Calibration of scale

| Scale reading | Microhenries | Scale deflection | Microhenries |
|---|---|---|---|
| + 49·74 | + 6158·0 | 99·52 | 12271·5 |
| − 49·78 | − 6113·5 | | |
| + 20·00 | + 2492·5 | 39·98 | 4931·5 |
| − 19·98 | − 2439·0 | 59·54 | 7340·0 |

Result 123·28 $\mu$H./cm. at readings ± 35 cm. of scale.

Table 9. Value of $M/Cc$

$\mu$H./cm. = 123·28 (table 8); $T_1 = 16{\cdot}571$ sec.

| Field coil current $C$ (A.) | Oscillating coil current $c$ (A.) | Deflections in cm. due to reversal from $+c$ to $-c$ | $M/Cc$ ($\mu$H./A.$^2$) |
|---|---|---|---|
| 1·4500 | 0·0018567 | 91·28 | 2089900 |
| 1·4500 | 0·0014610 | $71{\cdot}82_5$ | 2089900 |
| 1·3500 | 0·0015728 | 72·00 | 2090200 |
| 1·3500 | 0·0018910 | $86{\cdot}56_5$ | 2090200 |
| 1·2500 | 0·0016618 | 70·45 | 2090500 |

Mean result: $2{\cdot}090_1 \times 10^9$ cm./A.$^2$

Table 10. Logarithmic decrements and value of resistance

$M/Cc = 2{\cdot}090_1 \times 10^9$ cm./A.$^2$; $T_1 = 16{\cdot}571$ sec.; $T_0 = 16{\cdot}563$ sec.; $\lambda_0$ (common) = $0{\cdot}00088_0$.

| $C$ | $\lambda$ (common) | $\lambda(1-\beta)-\lambda_0$ | $R \times 10^9$ (c.g.s.) | Resistance by comparison (ohms) |
|---|---|---|---|---|
| | | | | 19·842 |
| 1·4500 | 0·05818 | $0{\cdot}1318_2$ | $19{\cdot}84_6$ | |
| 1·4500 | 0·05815 | $0{\cdot}1317_4$ | $19{\cdot}85_8$ | |
| | | | | 19·853 |
| 1·4500 | 0·05811 | $0{\cdot}1316_6$ | $19{\cdot}87_0$ | |
| 1·4500 | $0{\cdot}05807_1$ | $0{\cdot}1315_7$ | $19{\cdot}88_3$ | |
| | | | | 19·862 |
| 1·3500 | 0·05048 | $0{\cdot}1141_3$ | $19{\cdot}86_9$ | |
| | | | | 19·875 |
| 1·3500 | 0·05048 | $0{\cdot}1141_3$ | $19{\cdot}86_9$ | |
| | | | | 19·888 |
| 1·2500 | 0·04336 | 0·09776 | $19{\cdot}88_7$ | |
| 1·2500 | $0{\cdot}04332_5$ | 0·09768 | $19{\cdot}90_3$ | |
| | | | | $19{\cdot}906_5$ |
| 1·2500 | $0{\cdot}04326_5$ | 0·09754 | $19{\cdot}93_2$ | |
| | | | | 19·924 |

Owing to the fact that logarithmic decrements play such an important part in the method, we illustrate the accuracy attained and the method of evaluation by giving two examples recorded in table 11 below.

It should be pointed out that a logarithmic decrement worked out thus by differences inevitably yields the same result as if the slope of the corresponding graph were evaluated by zero sum or by the method described by Awbery*.

## § 6. SUGGESTIONS FOR FURTHER DEVELOPMENT

Though we have attempted to show both theoretically and practically the possibilities of the method, much on both sides remains to be done. Owing to our experimental conditions and the expenses involved in the construction of coils and other accessories, we would gladly see the method pushed to its limits in the hands of other observers.

* *Proc. Phys. Soc.* **41**, 384 (1929).

We believe that the measurement of logarithmic decrements, which depends, as such dynamic observations must, on the skill and form of the observer, should be carried out photographically, and some experiments made in this direction by Mr H. G. Bell strongly confirm this view. Periods also should be taken photographically by a running film in conjunction with a time trace, so as not only to save delay at a time when variation in the temperature and hence in the period would affect the accuracy of the experiment, but also to deal effectively and simply with the problem of self-inductance.

Table 11, Evaluation of logarithmic decrements

| Oscillating coil $C_2$ $C = 1{\cdot}36$ A. | | | | | Oscillating coil $C_3$ $C = 1{\cdot}45$ A. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Scale readings | | Amplitudes | Mantissa of logarithm | Differences of six swings | Scale readings | | Amplitudes | Mantissa of logarithm | Differences of four swings |
| | 49·40 | — | — | — | | 44·80 | — | — | — |
| 45·28 | | 94·68 | 0·97626 | — | 39·11 | | 83·91 | 0·92381 | — |
| | 40·10 | 85·38 | 0·93136 | — | | 34·30 | 73·41 | 0·86576 | — |
| 36·89 | | 76·99 | 0·88643 | — | 29·91 | | 64·21 | 0·80760 | — |
| | 32·51 | 69·40 | 0·84136 | — | | 26·24 | 56·15 | 0·74935 | — |
| 30·08 | | 62·59 | 0·79651 | — | 22·88 | | 49·12 | 0·69126 | 0·23255 |
| | 26·33 | 56·41 | 0·75136 | — | | 20·09 | 42·97 | 0·63317 | 0·23259 |
| 24·52 | | 50·85 | 0·70629 | 0·26997 | 17·49 | | 37·58 | 0·57496 | 0·23264 |
| | 21·32 | 45·84 | 0·66124 | 0·27012 | | 15·38 | 32·87 | 0·51680 | 0·23255 |
| 20·00 | | 41·32 | 0·61616 | 0·27027 | | | | | |
| | 17·26 | 37·26 | 0·57124 | 0·27012 | | | | | |
| 16·34 | | 33·60 | 0·52634 | 0·27017 | | | | | |
| | 13·95 | 30·29 | 0·48130 | 0·27006 | | | | | |
| Average for 6 swings = 0·27012<br>Whence $\lambda$ (common) = 0·04502 | | | | | Average for 4 swings = 0·23258<br>Whence $\lambda$ (common) = $0{\cdot}058146$ | | | | |

Large field-coils should be used, giving greater available sensitivity and allowing of more clearance between themselves and the oscillating coil. Further, direct steps should be taken to keep the resistance of the oscillating coil as constant as possible. With suitable field coils, the use of manganin for the oscillator is by no means out of the question.

We are meanwhile investigating practically how close to the linear law of inductance we can approach with a multiple-layered oscillating coil of slightly larger mean radius, as in our main research we were hampered by lack of clearance (owing to the sphere and suspension terminals) and so our oscillating coils were rather on the small side.

If the lower suspension is replaced by a mercury contact, the apparatus makes possible the determination, not only dynamically but also statically, of the rigidity of the main suspending wire, for it enables a constant and easily measurable couple to be applied electrically.

§ 7. ACKNOWLEDGMENTS

We would express our gratitude to Prof. A. Griffiths of Birkbeck College, who has done everything possible to encourage us in our difficulties and to provide us with necessary apparatus. We are also grateful to Mr H. G. Bell and to Mr F. Staley of Birkbeck College, on whose mechanical skill we have relied throughout the investigation, and to Mr R. A. Dobb, who has facilitated our work in many ways. We are indebted to Mr R. Belshaw for calculating the Legendre functions and for checking our other tables of special functions, and to Prof. G. Temple of the Imperial College of Science, who has taken great interest in our work and ever been ready to advise us, and to Dr R. G. Cooke of Birkbeck College who has made suggestions to us for further development on the mathematical side.

## APPENDIX: TABLES OF FUNCTIONS FOR CALCULATING THE MUTUAL INDUCTANCE FOR SPECIAL CASES OF NON-COPLANAR CIRCLES.

Table 12 gives the values and their logarithms of Legendre functions $P_n(\cos\theta)$, where $n$ is an odd integer, up to $n = 21$ for the angles 89° to 83° inclusive; and of $P_n(\cos\theta)$ and $P_n'(\cos\theta)$ up to $n = 31$ for the angle 79°. The values up to $n = 7$ are those of Tallqvist*. All higher values were specially calculated for us by Mr R. Belshaw in the first place by recurrence formulae, and checked independently by him and by us by the direct expression

$$P_n(\cos\theta) = \frac{4}{\pi}\frac{2\cdot4\ldots2n}{3\cdot5\ldots(2n+1)}\left[\frac{\cos(n\theta-\phi)}{(2\sin\theta)^{\frac{1}{2}}} + \frac{1^2}{2(2n+3)}\frac{\cos(n\theta-3\phi)}{(2\sin\theta)^{\frac{3}{2}}} + \frac{1^2\cdot3^2}{2\cdot4(2n+3)(2n+5)}\frac{\cos(n\theta+5\phi)}{(2\sin\theta)^{\frac{5}{2}}} + \ldots\right]$$

where $\phi = \theta/2 - \pi/4$.

Table 13 gives the logarithms of functions used in calculating mutual inductance from the expressions of equations (4), (5) and (8) of this paper. The functions of which the logarithms are tabulated are:

$$B = \frac{2}{n(n+1)}\frac{1^2\cdot3^2\cdot5^2\ldots n^2}{2^2\cdot4^2\ldots(n-1)^2},$$

$$C = \frac{2}{n(n+1)}\frac{1^3\cdot3^3\cdot5^3\ldots n^3}{2^3\cdot4^3\ldots(n-1)^3},$$

$$D = 1 + (n+2)(n-1),$$

$$E = 1 + 10(n+2)(n-1) + (n+4)(n+2)(n-3)(n-1),$$

$$F = 1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-3)(n-1) + (n+6)(n+4)(n+2)(n-5)(n-3)(n-1).$$

* *Acta Societatis Scientiarum Fennicae*, Helsingfors, **33**, 4 (1905), and **33**, 9 (1906).

Table 12. Values of spherical functions

| $n$ | $P_n$ (cos 89) | log $P_n$ (cos 89) | $P_n$ (cos 88) | log $P_n$ (cos 88) |
|---|---|---|---|---|
| 1 | + 0·0174524064 | $\bar{2}$·2418553 | + 0·0348994967 | $\bar{2}$·5428192 |
| 3 | − 0·0261653202 | $\bar{2}$·4177260 | − 0·0522429783 | $\bar{2}$·7180279 |
| 5 | + 0·0326767619 | $\bar{2}$·5142390 | + 0·0650650303 | $\bar{2}$·8133476 |
| 7 | − 0·0380725550 | $\bar{2}$·5806120 | − 0·0755080389 | $\bar{2}$·8779932 |
| 9 | + 0·0427577 | $\bar{2}$·6310143 | + 0·0843585 | $\bar{2}$·9261288 |
| 11 | − 0·0469330 | $\bar{2}$·6714783 | − 0·0919990 | $\bar{2}$·9637831 |
| 13 | + 0·0507147 | $\bar{2}$·7051339 | + 0·0986456 | $\bar{2}$·9940777 |
| 15 | − 0·0541767 | $\bar{2}$·7338125 | − 0·1044325 | $\bar{1}$·0188357 |
| 17 | + 0·0573691 | $\bar{2}$·7586781 | + 0·1094486 | $\bar{1}$·0392102 |
| 19 | − 0·0603276 | $\bar{2}$·7805160 | − 0·1137562 | $\bar{1}$·0559751 |
| 21 | + 0·0630786 | $\bar{2}$·7998820 | + 0·1174010 | $\bar{1}$·0696718 |
| $n$ | $P_n$ (cos 87) | log $P_n$ (cos 87) | $P_n$ (cos 86) | log $P_n$ (cos 86) |
| 1 | + 0·0523359562 | $\bar{2}$·7188002 | + 0·0697564737 | $\bar{2}$·8435845 |
| 3 | − 0·0781455571 | $\bar{2}$·8929043 | − 0·1037861291 | $\bar{1}$·0161393 |
| 5 | + 0·0968786895 | $\bar{2}$·9862283 | + 0·1278363599 | $\bar{1}$·1066544 |
| 7 | − 0·1116796606 | $\bar{1}$·0479741 | − 0·1459810293 | $\bar{1}$·1642964 |
| 9 | + 0·1236765 | $\bar{1}$·0922872 | + 0·1596458 | $\bar{1}$·2031575 |
| 11 | − 0·1334045 | $\bar{1}$·1251705 | − 0·1694988 | $\bar{1}$·2291666 |
| 13 | + 0·1411603 | $\bar{1}$·1497125 | + 0·1759198 | $\bar{1}$·2453147 |
| 15 | − 0·1471279 | $\bar{1}$·1676950 | − 0·1791658 | $\bar{1}$·2532551 |
| 17 | + 0·1514333 | $\bar{1}$·1802214 | + 0·1794429 | $\bar{1}$·2539263 |
| 19 | − 0·1541715 | $\bar{1}$·1880041 | − 0·1769401 | $\bar{1}$·2478263 |
| 21 | + 0·1554214 | $\bar{1}$·1915108 | + 0·1718480 | $\bar{1}$·2351445 |
| $n$ | $P_n$ (cos 85) | log $P_n$ (cos 85) | $P_n$ (cos 84) | log $P_n$ (cos 84) |
| 1 | + 0·0871557427 | $\bar{2}$·9402960 | + 0·1045284633 | $\bar{1}$·0192346 |
| 3 | − 0·1290784997 | $\bar{1}$·1108539 | − 0·1539374478 | $\bar{1}$·1873443 |
| 5 | + 0·1576637203 | $\bar{1}$·1977318 | + 0·1860957742 | $\bar{1}$·2697365 |
| 7 | − 0·1778359544 | $\bar{1}$·2500195 | − 0·2067077744 | $\bar{1}$·3153568 |
| 9 | + 0·1912894 | $\bar{1}$·2816909 | + 0·2177447 | $\bar{1}$·3379476 |
| 11 | − 0·1988401 | $\bar{1}$·2985040 | − 0·2202513 | $\bar{1}$·3429185 |
| 13 | + 0·2010073 | $\bar{1}$·3032118 | + 0·2150316 | $\bar{1}$·3325023 |
| 15 | − 0·1982155 | $\bar{1}$·2971376 | − 0·2028794 | $\bar{1}$·3072379 |
| 17 | + 0·1908789 | $\bar{1}$·2807579 | + 0·1846633 | $\bar{1}$·2663806 |
| 19 | − 0·1794384 | $\bar{1}$·2539154 | − 0·1613523 | $\bar{1}$·2077752 |
| 21 | + 0·1643764 | $\bar{1}$·2158395 | + 0·1340115 | $\bar{1}$·1271421 |
| $n$ | $P_n$ (cos 83) | log $P_n$ (cos 83) | | |
| 1 | + 0·1218693434 | $\bar{1}$·0858945 | | |
| 3 | − 0·1782789647 | $\bar{1}$·2511001 | | |
| 5 | + 0·2128790434 | $\bar{1}$·3281329 | | |
| 7 | − 0·2321080669 | $\bar{1}$·3656902 | | |
| 9 | + 0·2382868 | $\bar{1}$·3771000 | | |
| 11 | − 0·2328666 | $\bar{1}$·3671072 | | |
| 13 | + 0·2172037 | $\bar{1}$·3368672 | | |
| 15 | − 0·1927956 | $\bar{1}$·2850971 | | |
| 17 | + 0·1613402 | $\bar{1}$·2077426 | | |
| 19 | − 0·1247175 | $\bar{1}$·0959274 | | |
| 21 | + 0·0849331 | $\bar{2}$·9290770 | | |
| $n$ | $P_n$ (cos 79) | log $P_n$ (cos 79) | $P_n'$ (cos 79) | log $P_n'$ (cos 79) |
| 1 | + 0·1908089954 | $\bar{1}$·2805988 | + 1·0000000000 | Zero |
| 3 | − 0·2688460236 | $\bar{1}$·4295036 | − 1·2269394546 | 0·0888231 |
| 5 | + 0·2989725190 | $\bar{1}$·4756313 | + 0·971481534 | $\bar{1}$·9874346 |
| 7 | − 0·2913338271 | $\bar{1}$·4643909 | − 0·315154223 | $\bar{1}$·4985231 |
| 9 | + 0·2526096 | $\bar{1}$·4024498 | − 0·614849037 | $\bar{1}$·7887685 |
| 11 | − 0·1905053 | $\bar{1}$·2799071 | + 1·641528 | 0·2152483 |
| 13 | + 0·1141450 | $\bar{1}$·0574569 | − 2·562565 | 0·4086749 |
| 15 | − 0·0334361 | $\bar{2}$·5242156 | + 3·183962 | 0·5029679 |
| 17 | − 0·0419178 | $\bar{2}$·6223985 | − 3·354408 | 0·5256158 |
| 19 | + 0·1034638 | $\bar{1}$·0147884 | + 2·993787 | 0·4762209 |
| 21 | − 0·1449163 | $\bar{1}$·1611172 | − 2·110615 | 0·3244090 |
| 23 | + 0·1628042 | $\bar{1}$·2116656 | + 0·805062 | $\bar{1}$·9058293 |
| 25 | − 0·1567627 | $\bar{1}$·1952428 | + 0·743397 | $\bar{1}$·8712208 |
| 27 | + 0·1294338 | $\bar{1}$·1120478 | − 2·30214 | 0·3621317 |
| 29 | − 0·0859955 | $\bar{2}$·9344757 | + 3·62147 | 0·5588849 |
| 31 | + 0·0333865 | $\bar{2}$·5235709 | − 4·47502 | 0·6507949 |

A function $A$ used in equation (7) is defined thus:

$$A = \frac{2}{n(n-1)} \frac{1.3.5 \ldots n}{2.4 \ldots (n-1)},$$

and is easily deduced from the relation

$$\log A = 2 \log B - \log C.$$

In every case $n$ is an odd integer.

Table 13. Logarithms of special functions

| $n$ | log $B$ | log $C$ | log $D$ | log $E$ | log $F$ |
|---|---|---|---|---|---|
| 1 | Zero | Zero | Zero | Zero | Zero |
| 3 | $\bar{1}$·5740313 | $\bar{1}$·7501225 | 1·0413927 | 2·0043214 | 2·9595184 |
| 5 | $\bar{1}$·3699113 | $\bar{1}$·6429126 | 1·4623980 | 2·8048697 | 4·3051148 |
| 7 | $\bar{1}$·2327381 | $\bar{1}$·5726861 | 1·7403627 | 3·4649364 | 5·1750596 |
| 9 | $\bar{1}$·1289887 | $\bar{1}$·5200892 | 1·9493900 | 3·8890214 | 5·8196025 |
| 11 | $\bar{1}$·0454426 | $\bar{1}$·4779359 | 2·1172713 | 4·2279124 | 6·3322427 |
| 13 | $\bar{2}$·9754694 | $\bar{1}$·4427248 | 2·2576786 | 4·5105584 | 6·7588045 |
| 15 | $\bar{2}$·9152560 | $\bar{1}$·4124746 | 2·3783979 | 4·7531616 | 7·1243848 |
| 17 | $\bar{2}$·8624037 | $\bar{1}$·3859512 | 2·4842998 | 4·9657519 | 7·4444134 |
| 19 | $\bar{2}$·8153037 | $\bar{1}$·3623323 | 2·5786392 | 5·1549866 | 7·7290792 |
| 21 | $\bar{2}$·7728239 | $\bar{1}$·3410418 | 2·6637009 | 5·3255177 | 7·9854753 |
| 23 | $\bar{2}$·7341371 | $\bar{1}$·3216602 | 2·7411516 | 5·4807268 | 8·2187430 |
| 25 | $\bar{2}$·6986204 | $\bar{1}$·3038722 | 2·8122447 | 5·6231511 | 8·4327315 |
| 27 | $\bar{2}$·6657928 | $\bar{1}$·2874350 | 2·8779470 | 5·7547435 | 8·6303987 |
| 29 | $\bar{2}$·6352752 | $\bar{1}$·2721574 | 2·9390198 | 5·8770400 | 8·8140677 |
| 31 | $\bar{2}$·6067637 | $\bar{1}$·2578863 | 2·9960737 | 5·9912708 | 8·9855969 |

## DISCUSSION

Prof. A. Griffiths. The work has been carried out under the very great difficulties imposed by the limited accommodation available at Birkbeck College. The method seems to me to be good, and I venture to hope that it will be tested to a higher degree of accuracy at some larger laboratory.

Dr G. Temple. The theory of this paper centres around the series

$$M = Gq \sum_{n=1}^{\infty} \frac{2}{n(n+1)} \left(\frac{s}{r}\right)^{n-1} P_n'(\cos\psi)\, P_n'(\cos\phi)\, P_n(\sin\theta) \quad \ldots\ldots(2),$$

and the derived series which form the coefficients of powers of $\theta$ in equations (5) and (8). Dr Nettleton has estimated the sums of these series by the direct calculation and addition of the first sixteen terms, i.e. up to the term for which $n = 31$, $n$ being an odd integer. It is therefore important to know an upper bound to the magnitude of the remainders.

These values can be obtained from the following inequality,*

$$|P_n^m(\cos\theta)| \,.\, \sin^m\theta < \frac{\Pi(n+m)}{\Pi(n)} \left\{\frac{4}{(n\pi\sin\theta}\right\}^{\frac{1}{2}},$$

* E. W. Hobson, *The Theory of Spherical and Ellipsoidal Harmonics*, §§ 200–202 (Cambridge University Press, 1931). There is a misprint in equation (29) of page 313.

where $0 < \theta < \pi$, $n - m + 1 \geqslant 0$, and $m$ is a positive integer or zero. $\Pi(n)$ is Gauss's $\Pi$-function which reduces to $n!$ when $n$ is integral. The particular cases of this inequality required in the discussion of the series (2) are

$$| P_n(\cos\theta) | < 2(n\pi \sin\theta)^{-\frac{1}{2}}$$

and $$| P_n'(\cos\theta) | . \sin\theta < 2(n+1)(n\pi \sin\theta)^{-\frac{1}{2}}.$$

Now, according to Hobson's definition of $P_n^m(\cos\theta)$,*

$$P_n^1(\cos\theta) = -\sin\theta . P_n'(\cos\theta).$$

Hence $$| P_n^1(\cos\theta) | < 2(n+1)(n\pi)^{-\frac{1}{2}} \sin^{-\frac{5}{2}}\theta.$$

It follows that if $a_n$ is the term of order $n$ in the series (2), then $a_n$

$$| a_n | < C_n . n^{-\frac{3}{2}} . x^{n-1},$$

where $x = s/r$, $x$

and $C_n = 8(n+1) n^{-1} \pi^{-\frac{3}{2}} . \sin^{-\frac{5}{2}}\psi . \sin^{-\frac{5}{2}}\phi . \cos^{-\frac{1}{2}}\theta.$ $C_n$

Hence the magnitude of the remainder $R_N$, where $R_N$

$$R_N = \sum_{n=N}^{\infty} a_n,$$

is less than $$C_N \sum_{n=N}^{\infty} n^{-\frac{3}{2}} x^{n-1}.$$

It may similarly be shown that the estimation of the remainders in the series which form the coefficients of $\theta$, $\theta^3$ and $\theta^5$ in equation (8) depends upon allied series of the form

$$\sum_{n=N}^{\infty} n^{-s} x^{n-1},$$

where $s = \frac{1}{2}, -\frac{3}{2}$ and $-\frac{7}{2}$.

The order of magnitude of the sums of these series can be inferred from the Eulerian integral

$$\Pi(s-1) = \int_0^{\infty} t^{s-1} e^{-t}\, dt$$

if $s > 0$, or from Hankel's contour integral for the gamma function if $s < 0$. We have that

$$\Pi(s-1) . n^{-s} x^n = \int_0^{\infty} t^{s-1} (xe^{-t})^n\, dt,$$

whence
$$\Pi(s-1) \sum_{n=N}^{\infty} n^{-s} x^n = \int_0^{\infty} t^{s-1} \sum_{n=N}^{\infty} (xe^{-t})^n\, dt$$
$$= \int_0^{\infty} \frac{t^{s-1} (xe^{-t})^N}{1 - (xe^{-t})^2}\, dt,$$

since $0 < x < 1$, and the summation is over odd values of $n$.

Now $$1 - (xe^{-t})^2 \geqslant 1 - x^2.$$

Hence
$$\Pi(s-1) \sum_{n=N}^{\infty} n^{-s} x^n \leqslant \int_0^{\infty} \frac{t^{s-1} (xe^{-t})^N}{1 - x^2}\, dt$$
$$= \frac{\Pi(s-1)}{1 - x^2} . N^{-s} x^N.$$

* Equation (4), page 90.

Therefore $$\sum_{n=N}^{\infty} n^{-s} x^n < \frac{x^N}{(1-x^2) N^s},$$

and the magnitude of the remainder $R_N$ is less than

$$\frac{C_N \cdot x^{N-1}}{(1-x^2) N^{\frac{1}{2}}}.$$

Dr D. Owen. The authors have in this paper presented the Society with the results of a very careful and valuable piece of work. It is interesting to observe how the mutual inductance of a pair of coils can by proper choice of dimensions be made a linear function of their angular position. But the essential feature of the paper lies of course in the transformation, with undoubted gain, of the original Weber damping method. It is amazing, on the face of it, that a method depending on two dynamical observations, namely, of a time of oscillation and a logarithmic decrement, should be in the running for a high place among the means of accurate absolute determination of resistance. In this paper we see what good results may be obtained with no specialized experimental facilities at disposal.

Looking at the figures I am puzzled to discover the records of the times of oscillation with the vibrating coil on closed circuit—only $\lambda$, but not $T$, appears to be given in tables 5, 6 and 10.

Mr J. Guild said that the logarithmic decrement depends on variable factors, such as barometric pressure and the properties of the suspension, which would need to be taken into account for precision measurements.

Dr L. Hartshorn (*communicated*): The work described is very interesting as extending the range of possibilities of the method of measuring electrical resistance by observation of the damping of mechanical vibrations; and the question is revived whether, with the elaboration outlined in the paper, the method could compete with others for the absolute measurement of resistance. From experience of precision measurements of the kind involved, I cannot help feeling that the method of damping suffers from severe disadvantages, even in the improved form described. At the National Physical Laboratory, precision measurements have been made by the Lorenz method and by Albert Campbell's latest method. The Lorenz method requires only a mutual inductance and a speed of rotation, while the Campbell method requires only two mutual inductances, which can be equal, a frequency, and the ratio of two resistances. As against these, the present method requires two logarithmic decrements, a period of oscillation, a mutual inductance, and the ratio of two currents which cannot be observed simultaneously. No deflections have to be observed in either the Lorenz or the Campbell methods: each is a null method, and the mutual inductance involved is that of a rigid system, instead of one with a comparatively delicate suspension. These points are probably sufficient to show that for standardizing work, in which an accuracy of a few parts in 100,000 is essential, the present method could hardly compete with the best available. Probably its greatest asset is the simplicity of the apparatus required for a determination with moderate accuracy, say to a few parts in 10,000.

AUTHORS' reply. We thank Prof. A. Griffiths for his kind remarks and Dr G. Temple for the trouble he has taken in investigating the rapidity of the convergence of the infinite series on which our calculations depend.

In reply to Dr Owen: the question of the possible accuracy of measurement of $(\lambda - \lambda_0)$ strikes admittedly at the root of the method. Unfortunately our oscillating coils, owing to slight warming from the current-bearing field-coils, were usually slowly rising in resistance and consequently $\lambda$ under the same current showed a slow and continuous fall. By changing from a higher to a lower current we often obtained periods of steady temperature, as will be seen in tables 5 and 6, and under such conditions our logarithmic decrements were usually consistent to 1 part in 5000. That logarithmic decrements are capable of measurement with an accuracy exceeding ours will be seen from the data of Dorn,* who was handicapped by the change of period with arc of oscillation. When a large number of readings are taken during a decrement the value may be estimated with great accuracy, the various methods of estimating graph-slope yielding identical results, as the abscissae are successive integers.

In reply to Mr Guild: while the constancy of external conditions he mentions would be most desirable in future work, we point out that the measurement of $\lambda$ takes only 3 or 4 min. and the resistance of the oscillating coil is compared both before and after with a standard resistance. The statical part of the experiment may, as was shown in the paper, be performed later.

We entirely agree with the communicated remarks of Dr L. Hartshorn and are under no delusion that the method could ever compete for precision with the methods he alludes to. We have merely improved a very old and interesting method by removing quantities exceptionally difficult to measure and corrections difficult to apply. Nevertheless, in view of the heaviness of the system and the remarkably close approach to true damped-simple-harmonic motion over considerable amplitude, and the fact that the resistance measured may be large, we confess, especially in the light of the results obtained by Dorn, that we hope that some day the method may be pushed to its limits under more favourable conditions on the lines we suggest. Apart altogether from the absolute measurement of resistance, the linear law obtained is of interest and there are, perhaps, other possibilities for such a mutual-inductance dynamometer.

* *Wied. Ann.* **36**, 22, 398 (1889).

# THE ELECTROSCOPE CAPACITY BALANCE

By E. S. BROWN, D.Sc., A.C.G.I.

*Communicated by Prof. T. H. Laby, October* 14, 1931. *Read in title, January* 15, 1932

*ABSTRACT.* Capacities of a few micromicrofarads are measured by means of two electroscopes connected in series with a source of e.m.f. and shunted by variable condensers. The intersection of the gold leaves lies on a straight line so long as the capacities of the condensers are equal. The unknown capacity is shunted upon one of the condensers, the capacity of which is then adjusted till the sum of the two capacities is equal to the capacity of the remaining condenser.

THIS apparatus was developed by the author for the rapid comparison of small capacities of the order of a few micromicrofarads. Ordinary alternating current from supply mains of any frequency is used, and the accuracy is not affected by any variations of frequency, wave-shape or voltage. The apparatus may be easily and cheaply constructed, most of the requisites being found in any electrical laboratory.

Figure 1 shows a diagram of connections. Two single-leaf gold-leaf electroscopes $A$ and $B$ are connected in series across the secondary of a small step-up transformer giving about 1000 volts. The crossed images of the gold leaves of $A$ and $B$ are optically projected on a vertical screen as shown in figure 2.

The gold-leaf terminals of the two electroscopes are connected together, and the cases of the instruments are connected to the terminals of the transformer. In parallel with electroscope $A$, which has its case earthed, is connected a screened standard graduated variable condenser $K_A$ having its screen connected to the earthed side of the electroscope $A$. In parallel with the electroscope $B$ a condenser $K_B$ is connected with its screen connected to the high tension terminal of the transformer. $K_B$ need not be graduated and is employed merely as a ballast condenser.

Any unknown capacity $X$ which is to be measured may be connected in parallel with $K_A$ or disconnected from it at will by means of a low-capacity switch. This may conveniently consist of a fine springy wire which may be caught on the terminal of $X$ or released by touching it with an ebonite rod so that it springs clear of $X$.

With the connections of figure 1, when the voltages across the two electroscopes are equal, while the total voltage is varied, the locus of the intersections of the crossed images of the leaves is a line shown on the screen in figure 2 and denominated the *equivolts line*. This line may be obtained by connecting the electroscopes in parallel and varying the applied potential-difference. When the image of the intersection of the leaves falls on the equivolts line the voltages applied to the electroscopes must be equal.

If this condition is satisfied when connections are made as in figure 1, the combined capacities of electroscope $A$ and its parallel condenser $K_A$ must be equal to the combined capacities of the electroscope $B$ and its parallel condenser $K_B$. The intersection of the leaves may then be moved to right or left of the equivolts line by alteration of either condenser, or can be set exactly on the equivolts line. It is then known that the voltages on both sides and hence the capacities on both sides are equal because of their series connection.

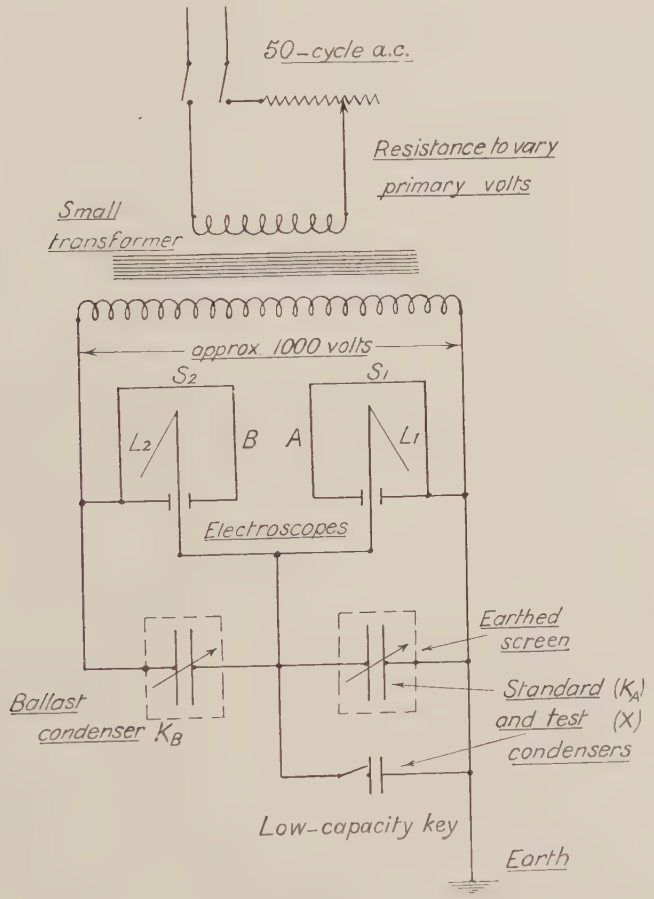


Fig. 1. Connection diagram.

The apparatus may be likened then to a balance with equal arms. The addition or removal, in parallel with the standard $K_A$, of any small capacity $X$ which is to be measured will disturb the balance, which may be restored by alteration of the variable standard condenser $K_A$. Thus the value of the small capacity may be read off by the change of reading of the standard required to restore balance.

As suggested by my assistant, Mr E. O. Willoughby, it is easy to obtain a number of cumulative readings of any very small capacity $X$ as follows. $X$ is

connected in parallel with $K_A$; $K_B$ is adjusted to bring the image of the leaves on to the equivolts line. $X$ is then disconnected and balance is restored by increasing the capacity of $K_A$ by an amount equal to $X$. Without any alteration of $K_A$, $X$ is now reconnected and $K_B$ is altered to restore balance again. $X$ having been disconnected once more, it is necessary to further increase the capacity of $K_A$ to restore balance, the increase giving a second value of $X$. This process may be carried on for any number of steps required. An average value of the capacity $X$ can then be obtained by dividing the total increase in the reading of the standard by the number of steps taken.

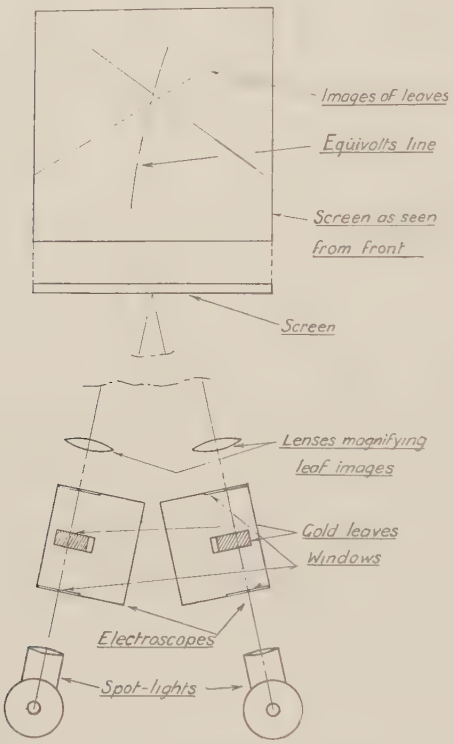


Fig. 2. Plan of optical system.

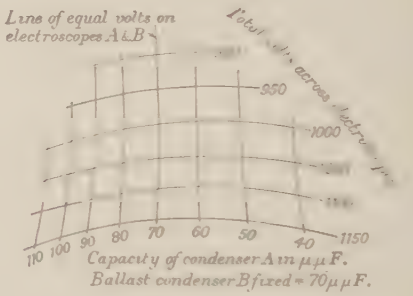


Fig. 3.

*Precautions for accuracy.* The insulation of the gold leaves, condensers and parts connected electrically thereto must be high to avoid errors due to leakage. Care must be taken to avoid hand and body-capacity effects on any parts of the leaf connections which are unshielded. Such effects may be at once detected by movement of the leaves. In order to get the greatest sensitivity the residual capacity of connecting wires, condensers and electroscopes should be kept as small as possible. It is suggested that as a safety measure a high resistance should be inserted in the lead from the h.t. terminal of the transformer.

The author considers that the apparatus described is of value for instructional purposes. Its use reveals to the experimenter and student the existence of small

inter-capacities which are often unsuspected or ignored, and facilitates their measurement. Many instructive and interesting experiments may be performed; the capacity of small wires, spheres, etc. suspended in air may be measured, and also the variation of these capacities due to distance from the earth or other bodies. The capacity of an average human being standing on an insulation stool about 5 ft. from the ground is found to be about $70\mu\mu$F. As the distance to earth has little effect, this value must approximate to the self-capacity of the human body.

The apparatus may be used for measuring the capacity of terminals or small switches, the residual capacity of microcondensers and the small capacities which are of importance in short-wave radio practice. It may also be used for rapid determination of specific inductive capacity. The author suggests that the apparatus may also be of value to manufacturers for rapid testing and sorting of small fixed condensers, the capacity of which is required to lie between certain limits. Application of the condenser terminals to a pair of contacts determines instantly whether the capacity lies between any desired limits; for these limits lines may be drawn in the diagram, between which the intersection of the leaves must lie; and further, if the screen be suitably graduated, the percentage error from the standard value may be instantly read off.

In conclusion, the author wishes to express his thanks to Mr E. O. Willoughby, who drew the diagrams and assisted in the experiments, and to Professor Laby for kindly communicating the paper.

# DEMONSTRATIONS

## "Apparatus for the measurement of relative humidity." *Demonstration given on December 4, 1931, by* Mr J. H. Orchard

While an immense amount of research work has been carried out on the problem of humidity-measurement, and many instruments have been devised, it appears to be generally considered there is still room for improvement in the apparatus for the determination of humidity on a commercial basis, and it is hoped that the instrument demonstrated is a step in the right direction. It uses the principle of the wet-and-dry-bulb platinum-resistance thermometer and has two main points of interest. The first is the form of thermometer adopted and the second is the means by which direct readings in percentage of relative humidity are secured without reference to humidity tables.

The thermometer consists of an enamelled and silk-insulated platinum wire, $0{\cdot}002_5$ in. in diameter, adjusted to have a fundamental interval of 40 ohms. This wire is doubled and drawn through copper capillary tubing, the bore of which is only sufficiently large to allow the doubled wire to pass without damage. The two ends of platinum wire are connected to small terminals moulded into a capsule and soldered on to one end of the capillary tubing, the other end of which is soldered up an inch or so beyond the looped end of the platinum wire. A thermometer so constructed has the following advantages: (i) it is hermetically sealed; (ii) it can be coiled into any shape; (iii) it can be copper or nickel-plated, or otherwise protected against corrosion; (iv) the platinum wire is not stressed and yet is entirely protected; (v) there is small heat-mass and large area, and thus a very small time-lag; (vi) the conduction error is extremely small.

To adapt this type of thermometer to the wet bulb of a hygrometer presented some difficulties, but in the model shown an artificial-silk sleeving has been slipped over the copper tubing, and the water is allowed to run down from an overhead supply instead of being drawn up by capillarity.

Immediately the preliminary difficulties had been overcome, this form of thermometer showed great advantages, for the purpose of humidity-measurement, over other forms with which I am familiar. The construction enables any number of wet-bulb thermometers to be made with the certainty that they will be identical as regards lag, self-heating and conduction error. The large area exposed for evaporation ensures that the thermometer is less subject than the majority to the effect of air-velocity, the depression not being affected above velocities of 1 m. sec. as compared with the more generally accepted figure of 3 m. sec. These remarks apply to the thermometer when the tubing is stretched out straight, which is the condition under which the writer has carried out all the tests illustrated on the records exhibited. When the thermometer is coiled up this good feature is naturally sacrificed, as a zone of humidity is formed which must be dispersed by the movement of air if true depression is to be obtained.

For the purpose of demonstration a portable instrument was shown, with a thermometer unit consisting of both wet-and-dry-bulb thermometers on one frame and without a water supply, the sleeving being wetted by submerging the wet bulb in water. The reading is obtained so quickly that under normal atmospheric conditions it is safe to rely on the wick remaining wet for a sufficiently long time. After the test position has been correctly set, to compensate for any variation of voltage in the dry cell, the switch is moved to the stud marked "Dry thermometer" and the large circular dial of the rheostat is set to the reading thus obtained. The switch is then moved to the humidity position and the percentage relative humidity is read directly on the lower of the two scales. The setting of the rheostat has adjusted the sensitivity of the galvanometer to that necessitated by the dry-bulb temperature, so that measurement of the depression of the wet bulb will enable relative humidity to be read direct. In the recording instrument this adjustment of the rheostat is done automatically by means of a relay mechanism.

As the actual contour of the humidity curve varies slightly with the dry-bulb temperature, and the rheostat can only be adjusted to suit one particular contour, an error is necessarily introduced by departure from the master curve selected. This error, however, is not greater than 2 per cent. relative humidity over a range of from 30 to 100° F.

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"Some effects produced when liquid jets of low velocity fall on a barrier." *Demonstration given on December* 4, 1931, *by* J. H. BRINKWORTH, A.R.C.S., D.I.C., D.Sc.

So very many investigations, both experimental and theoretical, have been made on liquid jets and their properties that it is, perhaps, rather surprising that the effects which are to be described have not been previously recorded. If the flow from a nozzle is restricted to such an extent that a continuous column of liquid is just obtained, and if this jet is allowed to fall on a barrier, it takes up a beaded form somewhat like a capillary surface of revolution. These beaded jets showing stationary vibrations are, in general, perfectly stable, provided that the barrier is such that the liquid can flow away freely from the region of impact of the jet, i.e. that the barrier is a stream-line surface for the source on its periphery.

By altering the position of the barrier the changes in form of the jet can be readily observed. In all the experiments shown the nozzle was a circular orifice of diameter about 1 cm., the barrier a glass sphere diameter 5 cm., and the flow of the order of 1·5 gm./sec.

It is of interest to compare the shape of these beaded jets in which a flow is maintained with the "strictly theoretical solutions of the configuration of a liquid under the influence of gravity and supported by a solid," i.e. under static-equilibrium conditions. Some of the latter were worked out graphically by Prof. J. Perry

about the year 1874, when he was a student under Lord Kelvin at Glasgow University. Illustrations of some of these original drawings will be found in Kelvin's *Popular Lectures and Addresses*, volume 1.

Figures 1 and 2 (*a*, *b*, *c*), reduced to the same nozzle diameter, illustrate the differences between the outlines of the theoretical static drop and the dynamical jet obtained under the experimental conditions referred to above.

For certain very definite positions of the barrier the jets exhibited in the present demonstration maintain vibrations transverse to the length of the jet. The outlines, figure 3 (*a*, *b*), show the minimum and maximum horizontal amplitudes of vibration in two cases. In other positions of the barrier the jet will show maintained longitudinal vibrations.

The thanks of the author are due to Mr F. C. Daniels for the excellence of figures 2 and 3.

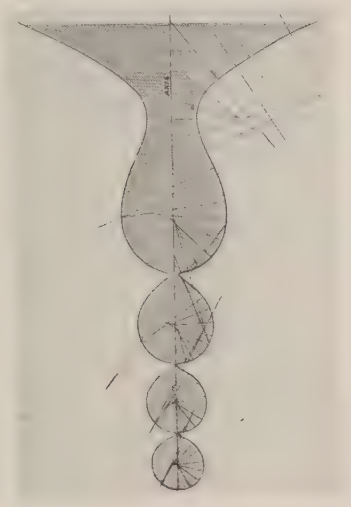


Fig. 1. Static.

Fig. 2 (*a*).

Fig. 2 (*b*).

Fig. 2 (*c*).

Fig. 3 (*a*).

Fig. 3 (*b*).

*To face page 226*

# REVIEWS OF BOOKS

*Problems of Modern Physics*, by H. A. Lorentz. Pp. vi + 312. (Boston, U.S.A.: Ginn and Co.) 12*s*. 6*d*.

The volume, published in 1927, is a record of lectures delivered by the late Prof. Lorentz in 1922, and naturally raises the question as to what may be called modern. We know that New College, Oxford, is still new after 550 years, and that the modern girl has been modern for a good while now. However, it seems clear that a time is ultimately reached at which *new* and *modern* cease to be applicable terms, as witness the case of the new woman. To what extent do these examples find their counterpart in physics? Is the subject of Lorentz's book the analogue of the new woman, with the new physics (born in 1925) taking the place of the modern girl? Both, at any rate, have been accused of a principle of indeterminacy, but there the resemblance appears to end.

The problems which Lorentz considers centre round two points, whether an accelerated electron must of necessity radiate, and whether light is corpuscular or a wave-motion.

The first problem has been solved, in the years which have passed since the lectures were delivered, by the expedient of abolishing the accelerated electron itself in the sense understood at that time; but, none the less, the discussion retains its interest.

In treating the second question, Lorentz comes very near to our present solution of the problem, when he refers to the possibility of considering Maxwell's electro-magnetic field as a ghost or shadow field, which serves to guide the photons in their motion. Unfortunately, he gets into difficulties in deciding how to apportion the energy between the field and the photons, and turns away to discuss other possibilities which might lead to solutions of the problem. This part of the work includes a very acute treatment of the classical theory of light as founded on Huyghen's principle, a section which is likely to be of permanent value.

I have said that the lectures group themselves about the discussion of two main problems, but it is natural that in the course of these discussions many short investigations should have to be carried out, and these are always done with great clearness. Their presence makes the book into a valuable reference work, as well as an exceptionally instructive example of how a master sets about solving a problem. Among these investigations are such problems as those of light-scattering, group-velocity, and a discussion as to whether the rotating-mirror method for the velocity of light measures group-velocity or wave-velocity.

Sandwiched between the main discussions, there is a delightfully clear account of the principles of the Einstein relativity theory. This is not a problem of modern physics in the sense of the other two matters dealt with, and (perhaps for that reason) it is likely to be of more permanent value to students.

No physicist who reads this book will waste his time in doing so, but perhaps the examination student had better wait till he has got his degree, since much of the material will not help in passing examinations.

It ought to be mentioned that a book-mark will be essential, because the volume is not divided into chapters, and does not indicate where one lecture ended and the next began. Moreover, the book-mark is required every few pages for another reason. Approximately half the book consists of notes which generally elaborate a mathematical argument merely sketched in the text; these notes are collected together at the end of the volume, making one's progress a somewhat jerky one. This, however, is a minor disadvantage,

and one which readers of the same author's *Theory of Electrons* have long ago learned to deal with.

The final word must be one of thanks to Prof. H. Bateman of the California Institute, without whose editorship it appears that the book might never have been published. He has added a few notes in places, in a noble effort to bridge the gap between 1922 and 1927.

J. H. A.

*The Combination of Observations*, by DAVID BRUNT. Pp. x + 239. (London: Cambridge University Press.) 12*s*. 6*d*.

Those of us who have, for some fifteen years past, industriously thumbed our copy of Mr Brunt's book know very well that it is one of the most reliable of its kind, written as it is by an author who combines wealth of knowledge with the teacher's instinct and is not afraid to begin at the beginning.

The subject is a fairly stable one, and much alteration in this new edition was neither to be expected nor desired. We find, therefore, the old bill of fare almost unaltered. The law of error; the case of one unknown; weighting; the adjustment of indirect observations containing several unknowns; most probable values and probable errors; criteria of rejection; alternatives to the normal law; correlation; harmonic analysis; the periodogram —these were, and are, the principal topics discussed. The chapter on alternatives to the normal law of error has been amplified, and the chapter on correlation now contains a section dealing with multiple correlation. The section dealing with harmonic analysis has been re-written, and the application of the method to the periodogram has been revised.

These are the principal changes to record, and it will be seen that they increase the usefulness without unduly enlarging the scope of the book, which remains one of the most useful elementary expositions of the orthodox theory in print.

The measure of Mr Brunt's orthodoxy may be gauged from the observation that the name of Campbell does not appear in his index of authors.

A. F.

*London and the Advancement of Science*, by various authors. Pp. vi + 321. (London: The British Association for the Advancement of Science.)

*The British Association—A retrospect*, 1831–1931, by O. J. R. HOWARTH, O.B.E., M.A., Secretary to the British Association. Pp. vii + 330, with 21 illustrations. (London: The British Association for the Advancement of Science.)

The first of these two books is the outcome of a happy suggestion by Dr Allan Ferguson that, instead of the usual handbook descriptive of the town and neighbourhood in which the Association is meeting, there should be prepared for the centenary meeting a book giving some indication of the part which London and London residents have played in the advancement of science. The field to be surveyed is truly stupendous and one cannot but admire the courage shown by the members of the Council when they decided to adopt the suggestion. The result is a book which rivals in usefulness and interest any previous publication of the Association. Considering further that the whole project was carried through in less than twelve months, we must proffer to all those concerned in the enterprise our very sincere congratulations.

The story opens with a very unorthodox survey by Dr Ferguson. This is written round the work of Chaucer, who is introduced to us in the unfamiliar garb of astronomer, Gilbert, Harvey, Bacon (an admirable summary here), Wren, John Hill, Henry Cavendish and William Hyde Wollaston. Varied fare, indeed, but one which the Londoner cannot but regard with mixed feelings. John Hill, for whom Dr Ferguson has obtained a testimonial

from Prof. G. T. R. Hill, was a person to provoke our interest rather than our pride. Neither his *Vegetable System* in 26 folio volumes nor his herb garden on the site of Lancaster Gate earned him the esteem of his contemporaries. Christopher Smart labelled him "Pimp! Poet! Paffer! 'Pothecary! Player!," but then Smart's feelings had been hurt. Johnson merely said of him that "he was an ingenious man, but had no veracity." A striking figure, well deserving of 9 columns in the *Dictionary of National Biography*, but not of a tenth of Dr Ferguson's survey.

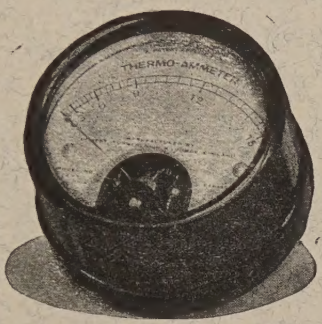
Cavendish, too, hardly shows the London scientist at his best. Certainly none of his predecessors had excelled him in experimental skill and few did as much to advance knowledge of physical science, but none was more indifferent to any desire the rest of humanity might have to share that knowledge. That his natural eccentricity was much increased by his daily diet of mutton we can well believe; but why give him six pages to the exclusion of men who gave the world the benefit of their talents?

What of that much maligned genius Robert Hooke; Boyle, who lived for 23 years in Pall Mall; Flamsteed, so ill-used (if we may believe his own account) by Newton; Halley, who first persuaded a British government to spend money on a scientific expedition; Samuel Molyneux, whose telescope at Kew revealed the aberration of light; Stephen Gray of Charterhouse; William Watson, once apothecary; John Canton, the Spital Square school-keeper; Stephen Hales, Teddington curate and founder of plant physiology? Perhaps, after all, Dr Ferguson was wise: the list is endless, space was short and time still shorter.

Following the introductory survey are two chapters descriptive of the formation and aims of all the more important scientific societies; an exceptionally interesting historical account of education in London by T. L. Humberstone; a chapter by Sir Frank Heath summarizing the activities of the Government in connexion with scientific research; an account of the development of medicine in London by H. H. Bashford; sections descriptive of the London museums past and present, of the Royal Observatory, Kew Gardens, and the John Innes Horticultural Institution; finally an all too brief *History of the London Makers of Scientific Instruments*. This last chapter contains many interesting facts, new at any rate to the reviewer, but is somewhat marred by the unaccountable absence of any reference to Thomas Tompion of Water Lane, Fleet Street, and his famous pupil George Graham, both buried in the same grave in Westminster Abbey.

There is little space left for notice of the second volume. It is indeed only a reprint of the *Retrospect*, published in 1922, but brought up to date, in particular, by the addition of a chapter describing Down House and its acquisition by the Association. It still contains, by implication, the statement that the late Sir Henry Roscoe was appointed President of the Association at the phenomenally early age of 5 years. D. O. W.

# TURNER

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